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Application of the one-parameter Bayesian method as the PBMD for concrete mix proportion design

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This paper presents a systematic approach for estimating material performance and designing the mix proportion of concrete based on an application of a Bayesian method in the form of satisfaction curves. The one-parameter satisfaction curve represents a satisfaction probability for a concrete performance criterion as a function of the concrete material parameters. An analysis method for combining multiple satisfaction curves to form one unique satisfaction curve that can reduce the performance of concrete to a single evaluating value, the goodness value, is proposed as an evaluation tool for the performance-based mixture design procedure. The proposed performance-based mixture design procedure is applied to test cases to obtain a target-oriented concrete mix proportion design and to verify the validity of the proposed method. Finally, the expected performance results of a concrete mix proportion designed using performance-based mixture design are compared with results calculated using the American Concrete Institute estimation equation to check whether the method is applicable to actual construction.

Introduction

The main goal of the paper is to present the application of a performance-based mixture design (PBMD) for concrete mix proportion design using satisfaction curves obtained from the one-parameter Bayesian method. As introduced in a previous paper (Kim *et al.*, 2009), PBMD is the application of the Bayesian method (Ang and Tang, 2006; Box and Tiao, 1992) and performance-based design (PBD) (Performance Based Building, 2005) for designing concrete mix proportions. The Bayesian method is a conditional-statistical method which was first applied to structural engineering by Shinozuka *et al.* (2000) and Singhal and Kiremidjian (1996, 1998). Generally, it is presented as the conditional probability of exceeding some limit state (i.e. collapse) for a given ground motion. In PBMD, this methodology has been used to assess concrete material performance on the basis of certain conditional parameters such as strength, workability and water-penetration depth, among others.

Because the current system uses a trial-and-error type of concrete mix proportion design, systemising the design process is difficult. However, owing to the increase in international commerce between countries with vastly different cultures, economical standards and technologies, the need for a systematic and general design concept such as PBMD has arisen. Current prescriptive design methods used by various national and regional codes are insufficient to develop this type of PBMD procedure. Therefore, a totally new concept of performance satisfaction based on selected material parameters must be introduced. In order to systemise the design procedure, a well-outlined general PBMD design procedure is proposed. The current practices for concrete mix design are based on workability and compressive strength performance, while durability is accounted for largely by prescriptive measures. Since a definite and general mathematical relationship involving variable factors and performance characteristics is not available, the procedures rely mostly on empirical tables and charts, among other things, which are used in subsequent test trials. The method proposed in this study will therefore require large-scale tests and trials.

The concept of Bayesian conditional probability has been used to develop the satisfaction curve, which is employed to evaluate the performance satisfaction probability of concrete material parameters. A satisfaction curve developed using the one-parameter Bayesian method introduced in a previous paper (Kim *et al.*, 2009) for a particular criterion value is obtained by computing the conditional probabilities within a realistic range for the criterion value of various concrete material parameters. The conditional probability is defined as

$$P_{ik} = P[S \geqslant s_i | Y = y_k] \tag{1}$$

where P_{ik} is the probability of exceeding the criterion level s_i for a given concrete material parameter y_k; S is the criterion random variable defined on the criterion level vector $S = \{s_0, s_1, \ldots, s_n\}$; and Y is the concrete material random variable. This equation indicates that the condition is specific to a particular criterion level.

In the present paper, a design procedure for PBMD with detailed methodologies, mainly composed of three main stages consisting of a total of seven steps, is introduced to show that the step-by-step mix proportion design procedure satisfies usage requirements. Within the scheme of the PBMD procedure, satisfaction curves are used as an evaluating tool to check whether the required criteria are satisfied. Some new concepts will be defined in this study, such as the interrelationship parameters of various performance levels of concrete and the importance factors of parameters obtained by determining whether the degree of each material parameter meets the target criterion. In addition, a method for combining multiple satisfaction curves using the concept of a goodness value, a common value for all concrete material parameter values, is proposed.

Finally, practical design examples are given to explain how PBMD is used in the design process. The analytical mixture design obtained from the PBMD method is then verified with experimental results to ensure the feasibility of practical applications. In addition, the analytical results are verified by comparing them with the results obtained using the method presented in ACI 214R-02 (ACI, 2002).

Design procedure of PBMD

The proposed PBMD procedure is a process for developing an optimal concrete mixture design using the satisfaction curve concept. The overall methodology of the PBMD procedure is shown in Figure 1 as a flowchart with three main stages composed of (*a*) the pre-design stage of making assumptions on the basis of a client's requests; (*b*) the evaluation stage of initial designing and optimisations; and (*c*) the comparison stage for intermediate modifications and final determination of the design. In the first step of the first stage, a design engineer must precisely understand a client's requests. In the second step, he or she makes initial decisions about material properties, designs the target criteria, selects the performance class of the concrete and establishes an initial standard for concrete mix proportion design. In the second stage, a performance evaluation of the initial design is performed by collecting data. Satisfaction curves are developed in the third step, and then multiple satisfaction curves are combined into a single satisfaction curve using the concepts of the importance factor and goodness value in the fourth step. The third and fourth stages verify that the standard concrete mix proportion satisfies the target performance criteria selected in the first step. If the mix proportion does not satisfy all of the performance criteria, the mix proportion has to be modified so that the final mix proportion satisfies the target performances. A comparison of the originally selected performance criteria with the actual evaluated performances, modifications and optimisations of the concrete mix proportion, and final verifications of a

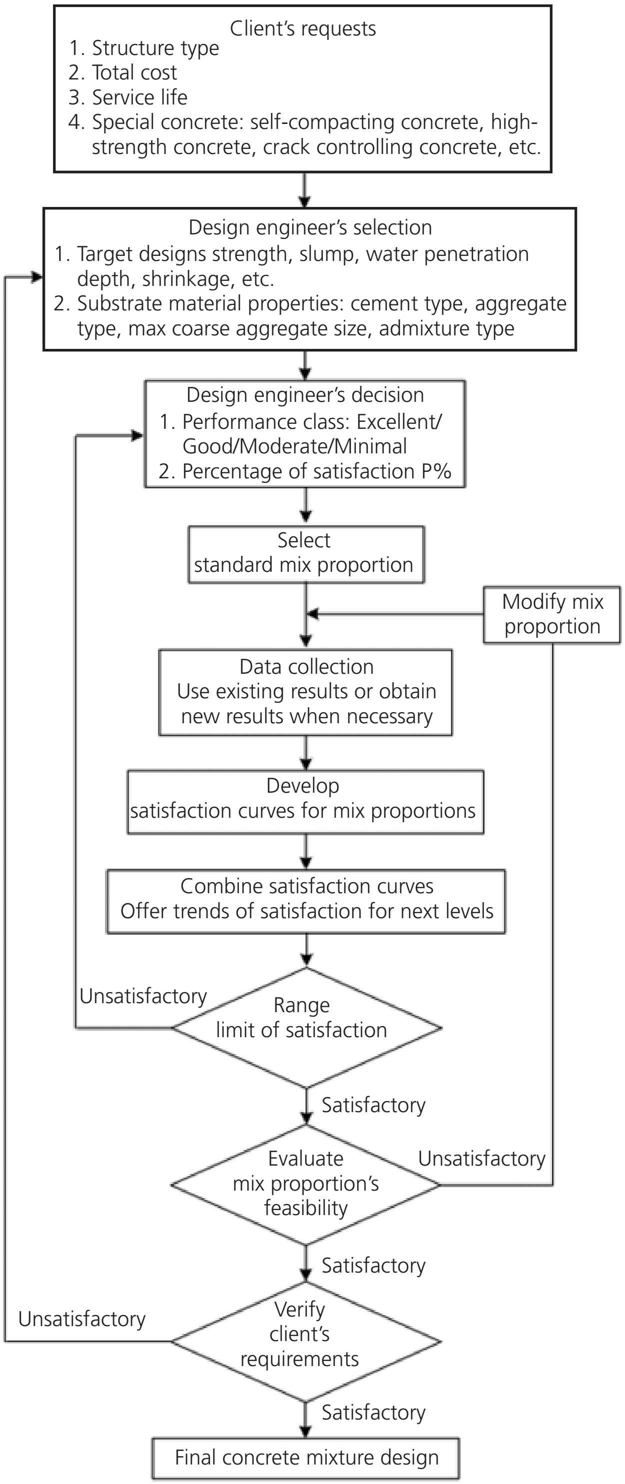


Figure 1. Main procedures of PBMD using the Bayesian method

client's requests are the three tasks carried out in the fifth, sixth and seventh steps, respectively, in the third stage. In all, the three stages have seven total steps. The PBMD procedure is explained in detail in the following sections.

Initial design decisions

Step 1 – client's requests

The PBMD's ultimate objective is to satisfy a client's requests for concrete material performance, which are used for the design and construction of the structure. In most cases, a client is focused mainly on broad criteria such as structure type (e.g. bridge, building, dam, etc.), total material cost and structure-service durability. In some special cases, a client might request special qualities in concrete, such as abundant workability for self-compacting concrete, high strength for prestressing required concrete and fibre concrete for crack-controlling concrete. According to a client's requests, the appropriate type of concrete with the requisite features has to be chosen. Therefore, a concrete designer has to clearly understand the client's requests and know what type of concrete will satisfy those requests before making initial selections regarding material parameters.

Step 2 – initial decisions and standard mixture design

According to the initial selection of concrete type, a design engineer selects substrate material properties (i.e. type of cement, aggregate, admixture etc.) for the design. In some cases, some special additives (e.g. fly ash (Malhotra, 1989), silica fume (Larrard *et al.*, 1992) and fibre (Naaman and Reinhardt, 1997) etc.) are chosen to satisfy the required performance. For example, in order to obtain high-strength concrete (HSC), a coarse aggregate should be strong and durable. In addition, the maximum coarse aggregate size should be smaller than that of normal-strength concrete (NSC), and a fine aggregate should have a fineness modulus greater than 3·2. Usually, one or more supplementary admixtures should be mixed (i.e. fly ash class C or F, ground-granulated blast furnace slag, silica fume, metakaolin natural pozzolanic materials, etc.) and the water-to-cement (w/c) ratio should be in the range 0·23 to 0·35. Moreover, other detailed conditions should be satisfied to achieve high-quality HSC (NRMCA, 2001).

Target design performance criteria, such as compressive strength, slump and water-penetration depth, among other criteria, must be decided. In addition, a design engineer should determine the performance class (i.e. excellent, good, moderate or minimal) of concrete material for the type of structure usage. For example, HSC (Holland *et al.*, 1988; Price and Hynes, 1996) used for prestressed concrete nuclear containment vessel structures, water anti-penetrating concrete (McCarter *et al.*, 1996; Vuorinen, 1985; Yousri, 2008) for offshore infrastructures, NSC for residential structures and sufficiently durable concrete for temporary storage structures can be assigned as materials that require excellent, good, moderate and minimal performances respectively. The performance class and compatible probability determine the difficulty of satisfying the client's request for concrete material. According to the target performance criteria, a standard concrete mix proportion is initially designed on the basis of conventional codes or standards. This initial standard mix proportion is considered to be a reference mix design. The optimal mix proportion which satisfies the required performance will be derived from this standard mix proportion.

Performance evaluation methodology

Step 3 – data collection and satisfaction curve development

To evaluate material performance, the data used to develop satisfaction curves must be compatible with the design engineer's intentions. In order systematically to evaluate and determine a mix proportion using the PBMD procedure, the relationships between the concrete material parameters and performance need to be considered as a hierarchy of needs ranging from concrete substrate materials to material properties to mechanical properties and all the way up to the cost and service life. The development of satisfaction curves requires the characterisation of concrete material parameters and the identification of different degrees of concrete material performance. The type, quantity and quality of substrate materials (i.e. water, cement, aggregate, admixture etc.) are important characteristics that affect final concrete performance. In this study, the recommended levels of material quality are shown in Figure 2 and are divided into five levels as follows

(*a*) level 1: the target level consisting of the cost and service life of the concrete

(*b*) level 2: the structure's material parameter levels, comprising safety, constructability, durability, and damageability

(*c*) level 3: the characteristic material performance levels, comprising strength, elastic modulus, workability, crack width, penetration depth, shrinkage, creep and so on.

(*d*) level 4: the material parameter level consisting of water content, cement content, w/c ratio, aggregate size and content, admixture content and so on

(*e*) level 5: the substrate level consisting of aggregate quality, cement type, admixture type and so on.

As shown in the hierarchy of the levels in Figure 2, clients are ultimately concerned about the cost and service life of the final mix proportion design, which are affected by factors in other lower levels. However, in this study, the 'substrate material' level will not be considered because the quality and types of substrates are assumed to be fixed. The relationship between concrete characteristic performance and concrete material parameters is established using a satisfaction curve obtained from the one-parameter Bayesian method. The possibility of exceeding the threshold criteria (i.e. target performance value) of each concrete mix proportion will be determined using the developed satisfaction curves.

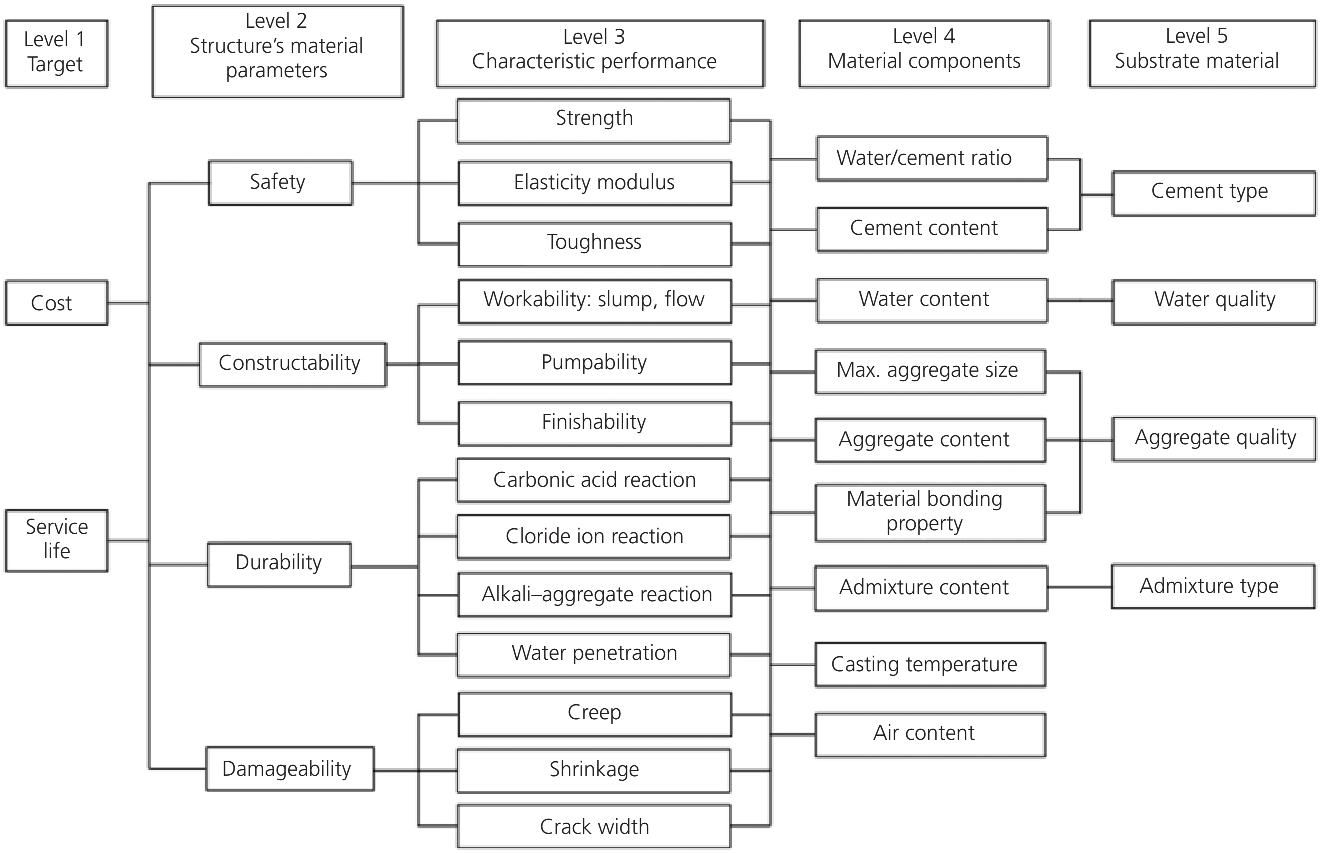


Figure 2. Parameters for various levels of concrete

Step 4 – satisfaction curve combination

When all of the satisfaction curves are developed from the required threshold criteria for a given mix proportion design, they are combined to develop a unified satisfaction curve for a mix proportion's threshold criteria. The unified satisfaction curve is expressed as the probability of performance satisfaction as a function of the goodness values. Through the use of the goodness value concept, multiple satisfaction curves are combined for each level to determine the satisfaction curve trend for that respective level, which will then be used to evaluate the overall satisfaction for the level under consideration. This process will be performed for all of the hierarchy levels until broad material levels of safety, constructability, durability and damageability are determined. In order to perform this combination process for achieving a unified satisfaction curve, the concepts of goodness value and importance factor are introduced as a way of accounting for the importance of each concrete parameter of the target performance as a quantified value and to harmonise the various parameter values (e.g. x-axis values in a satisfaction curve).

An example of the interrelationship between concrete strength performance and material component parameters representing the combination of satisfaction curves at level 4 for the level 3 categories is shown in Figure 3. The importance factor of each material parameter is determined by analysing the available data. These values are different for each threshold value and for each criterion. The parameters that greatly influence the criterion are considered to be the main parameters to which higher values are assigned. For example, the main parameters affecting workability are maximum size, grading, shape and the texture of the coarse aggregate as well as water content. Therefore, when the satisfaction curves are combined for workability performance, these material parameters will be given higher importance factor values than those of other parameters. As for the strength of concrete, the main parameters are w/c ratio, aggregate/cement ratio, aggregate properties, and the maximum coarse aggregate size. Nevertheless, in practice, the w/c ratio is the most significant parameter in determining the strength of concrete. The main parameters that influence shrinkage include aggregate content, w/c ratio and water content (Neville, 1995). Therefore, of the parameters affecting a particular performance, the importance factor values of the more important parameters are assigned higher values than those of the less important ones.

Since all parameter values and ranges are different, when combined they must be calibrated so that the x-axis parameter value

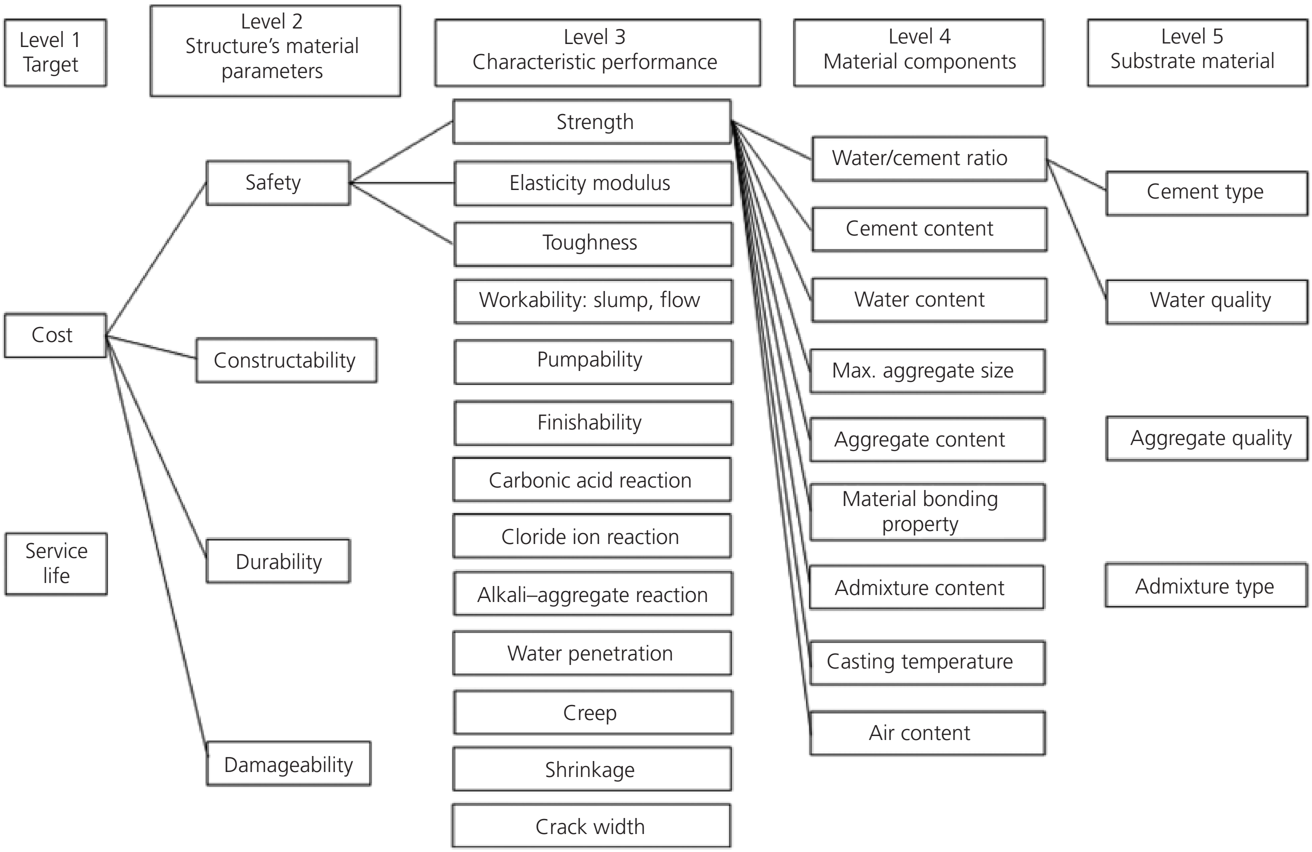


Figure 3. Relationships of the parameters to the levels of concrete

represents a common value in a combined satisfaction curve. In order to achieve this end, a calibrated parameter called the goodness value is introduced to represent and calibrate all concrete material parameter values (Figure 4). This combining method can be performed through the following refinements.

(*a*) Because a concrete mixture has many material parameters with varying ranges of values, it is important to calibrate the ranges such that the parameter values can be represented in compatible ranges from 0 to 1. In order to calibrate these parameters, the normalising method is used in the combining stage as a way to simplify the parameter ranges for design usage. After the normalisation step, the goodness value for each parameter is calculated as G_i as follows

$$G_i = \frac{X_i - X_{i\min}}{X_{i\max} - X_{i\min}} \tag{2}$$

where X_i is the concrete parameter and $X_{i\min}$ and $X_{i\max}$ are the minimum and maximum values of the limit range of the parameter X_i, respectively.

(*b*) To create an absolute parameter value as a reference value in a design, the minimum satisfaction requirement percentage is defined as the 'reference value.' For the purposes of a hypothetical case, all goodness values of the material parameters of the standard mixture proportion can be equilibrated to 0·5 by shifting the satisfaction curves in the *x*-axis direction to that reference point.

(*c*) Moreover, in performance-based seismic designs, the governing factor (i.e. the peak ground acceleration, PGA, for earthquakes) follows a certain trend such that the probability of damage is always increasing. However, in material design or characteristics, the global trend is not always formed in the same direction. For example, the increase in the w/c ratio in the normal mix proportion condition without any additives or admixtures results in decreased strength but increased workability. In order to combine the material parameter satisfaction curves into a single overall satisfaction curve for each level, the overall trends have to be compatible. In order to resolve this incompatibility, if the overall trend selected is a 'positive' trend, the opposite trend can be converted into a 'positive' trend by replacing the G_i value with $(1-G_i)$ and strictly maintaining the compatible probability P%.

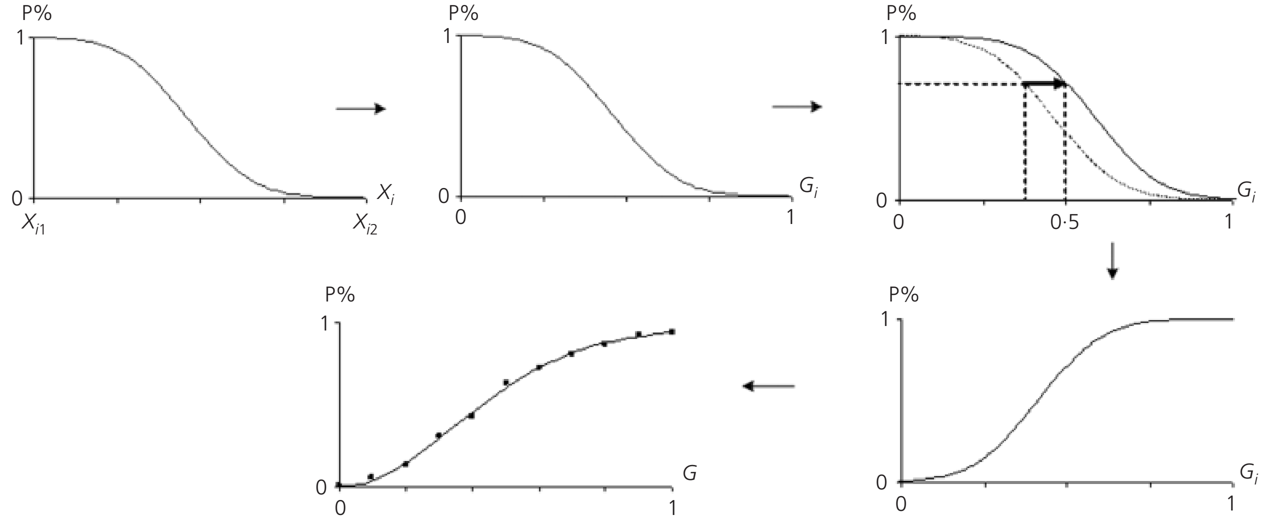


Figure 4. Combination of various concrete parameters

(*d*) The combination process is followed by calculating the probability at each goodness value as

$$P_j^k = P_i\alpha_i \tag{3}$$

where P_j^k is the probability value of criterion j at the considered level k; P_i is the probability of the material parameters; and α_i is the importance factor of the material parameters in the mixture design for a specified criterion.

Non-linear regression for a final satisfaction curve is performed to obtain a smooth curve of the combined data. The purpose of the combining process is to help design engineers clearly understand the performance trends at each level of the material hierarchy. In addition, another goal is to optimise the concrete mixture proportion according to the availabilities and qualities of substrates to supply the type of concrete needed for a required structure type according to the client's requirements and the structural usage.

Mix proportion design verification and modification

Step 5 – from original request to actual evaluation comparison

The performance satisfaction probabilities P% of the standard mix proportion designed according to the prescriptive code regulations are initially evaluated. The first step is to check whether the performance probability of the standard mix proportion is within the range of satisfaction. If the satisfaction probability deviates significantly from the required range of satisfaction (i.e. it is dissatisfying), the deviation indicates that the initially selected performance class and the corresponding probability satisfaction are unreasonable for this mix proportion with respect to the design usage. If this happens, then another performance class and corresponding satisfaction probabilities should be selected. The first design procedural loop is carried out to adjust the performance class and the corresponding probability satisfaction.

Step 6 – concrete mix proportion modification

If the condition in step 5 is satisfied, the performance probability of the standard mix proportion is within the range of required satisfaction. The next task is to determine which concrete material parameters do not satisfy the required performance satisfaction probability. Those parameters will be modified using developed satisfaction curves to obtain better probability values. The objective of the second design procedural loop is to modify the mix proportion to satisfy the required satisfaction probability.

Step 7 – client request verification for final concrete mix proportion design

The last condition that needs to be checked is whether the final modified concrete mix proportion satisfies the client's requests with respect to the satisfaction probability trends. If a client's requests are not completely satisfied, the target design criteria of the concrete material have to be modified because the satisfaction trend of the mix proportion design shows that it is deficient for construction usage. For example, Figure 5 shows two trends in the combined satisfaction curves for safety performance. In Figure 5, trend 2 is better than trend 1, because the initial half of the goodness value range of the satisfaction curve for trend 1 has a lower satisfaction probability than that of trend 2. This indicates that the mix proportion for trend 1 is significantly more dangerous than that for trend 2 with respect to material safety. Therefore, the initially chosen performance criteria will be modified to resolve this problem. This enhancement procedure is the most significant and is the last design procedural loop in the PBMD procedure. If differences between the old and new values

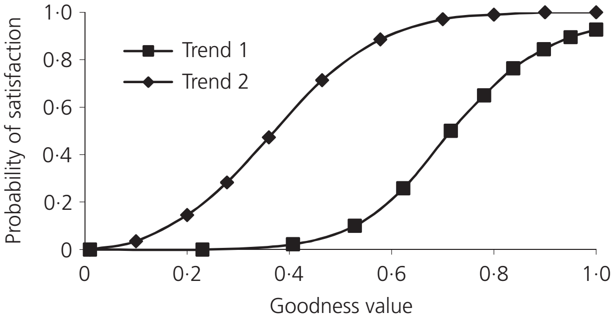


Figure 5. Safety trend in the satisfaction curve

of the target design criteria are significantly large, the data for the concrete mix proportion used to develop the satisfaction curves should be updated. Once the new initial design criteria are selected, then the PBMD procedure will start anew until a mix proportion is determined to be satisfactory, becoming a final concrete mix proportion design. The final concrete mix proportion obtained from the third and final design procedural loop will have to satisfy all of the client's requirements and must offer good trends in the satisfaction probability for site usage.

Design example

An example of applying the PBMD method to concrete mixture design using satisfaction curves to satisfy the required compressive strength and elastic modulus resulting in safety satisfaction is presented in this section. Some empirical formulae can be used to calculate the elastic modulus from the compressive strength. However, since this study's objective is to propose a new mix design method to obtain not only the required criterion values but also the satisfaction probability, a simple example is used to explain how the PBMD method can be applied to design a concrete mix proportion that satisfies the required safety performances of the compressive strength and the elastic modulus. The extensive examples concerning constructability, durability, and damageability need to be studied more rigorously in future research.

Initial design decisions

Structure type

A client's request for a concrete mix proportion in the construction of a new structure with a normal safety level requirement, such as simple housing usage and being replaceable after a moderate service life, would result in the selection of ordinary concrete requirements (e.g. not including any special requirements such as high water impenetrability, HSC or high corrosion preventability, among other requirements).

Substrate material selection

According to this underlying usage requirement, a design engineer decides to select ordinary substrate materials with the following material properties

Type I Portland cement conforming to KS L 5201:2006 (KSA, 2006) is used in all mixtures. The chemical and physical properties of the cement are presented in Tables 1 and 2, respectively.

The selected fine aggregate is natural river sand. The properties of the fine aggregate are determined and must fulfil the requirements of KS F 2526:2007 (KSA, 2007). Tables 3 and 4 present the properties of the sand and its gradation respectively.

Natural crushed stone aggregate with a maximum size of 25 mm and a bulk density of 1565 kg/m^3 is selected. Because this structure is an ordinary concrete structure, the use of special admixture materials such as slag, fly ash and admixture is unnecessary.

Compound	Abbreviation	Limit of KS L 5201 (KSA, 2006)
Silica	SiO_2	–
Alumina	Al_2O_3	–
Iron oxide	Fe_2O_3	$< 5.0\%$
Magnesia	MgO	$< 3.5\%$
Sulfite	SO_3	$< 3.0\%$
Loss of ignition		–
Tricalcium silicate	C_3S	–
Dicalcium silicate	C_2S	–
Tricalcium aluminate	C_3A	–
Tetracalcium aluminoferrite	C_4AF	

Table 1. Chemical composition of cement

Physic properties	Limit of KS L 5201 (KSA, 2006)
Finesse	$\geqslant$ 280 m^2/kg
Initial setting time	$\geqslant$ 60 min
Final setting time	$\leqslant$ 10 h
Soundness	$\leqslant$ 0·8%
Compressive strength at 3 days	$\geqslant$ 12·5 MPa
Compressive strength at 7 days	$\geqslant$ 22·5 MPa

Table 2. Physical properties of cement

Properties	Limit
Finesse modulus	2·8
Max. size	4·75 mm
Density	1693 kg/m^3
Specific gravity	2·62

Table 3. Properties of sand

Sieve size: mm	Accumulated percentage passing: %	Limit of KS A 5101–1 (KSA, 2004): %
4·75	96·5	95–100
2·36	87·2	80–100
1·18	78·6	50–85
0·60	55·7	25–60
0·30	24·5	10–30
0·15	3·8	2–10

Table 4. Gradation of fine aggregate

The initial compressive strength and elastic modulus design targets are 25 MPa and 25 GPa respectively. Other concrete material criteria such as water penetration depth and drying shrinkage strain are not important for this type of structure.

Performance class and compatible satisfaction probability

For this type of structure with a normal safety level, the performance class requirement for the concrete mix proportion with compressive strength and elastic modulus criteria of 25 MPa and 25 GPa, respectively, should be within a 'moderate performance class', equivalent to a required probability of approximately 65% for exceeding this criterion.

Standard mixture design

Based on the target design criteria, a standard mixture proportion is designed, and the compatible experimental results are collected for developing satisfaction curves. The aim of this step is to obtain data appropriate for a Bayesian probability analysis. The data can be of any type available, including analytical and/or experimental results or even previous design results.

According to ACI 211.1-91 (ACI, 1991), a standard mixture is designed to meet the requirements of 100 mm slump and 25 MPa compressive strength. This mixture is titled MSD in Table 5. More mix proportions are considered by varying one parameter (i.e. cement content, water content, maximum aggregate size etc.) while keeping the other parameters constant. Therefore, the interdependency of the material parameters was analysed to determine the independent contribution of a material parameter on the performance properties of the mix design. For example, MWC80W represents a mix proportion with a w/c ratio of 80% by varying the *w* parameter. These mix proportions are listed in Table 5. Three compressive strength and elastic modulus result pairs for each of the 19 mix proportions are shown in Table 6 and Table 7.

Symbol	Mix proportion					
	w/c: %	s/a: %	w: kg/m^3	c: kg/m^3	fa: kg/m^3	ca: kg/m^3
MSD	50	42	175·6	351·1	737·2	985·8
MWC80W	80	42	280·9	351·1	737·2	985·8
MWC80C	80	42	175·6	219·5	737·2	985·8
MSA35FA	50	35	175·6	351·1	540·0	985·8
MSA50FA	50	50	175·6	351·1	1000·0	985·8
MSA35CA	50	35	175·6	351·1	737·2	730·0
MSA50CA	50	50	175·6	351·1	737·2	1350·0
MWC35W	35	42	122·9	351·1	737·2	985·8
MWC65W	65	42	228·2	351·1	737·2	985·8
MWC35C	35	42	175·6	501·6	737·2	985·8
MWC65C	65	42	175·6	270·1	737·2	985·8
MWC45W	45	42	158·0	351·1	737·2	985·8
MWC55W	55	42	193·1	351·1	737·2	985·8
MWC45C	45	42	175·6	390·1	737·2	985·8
MWC55C	55	42	175·6	319·2	737·2	985·8
MSA39FA	50	39	175·6	351·1	640·0	985·8
MSA45FA	50	45	175·6	351·1	840·0	985·8
MSA39CA	50	39	175·6	351·1	737·2	870·0
MSA45CA	50	45	175·6	351·1	737·2	1100·0

w, water content; c, cement content; fa, fine aggregate; ca, coarse aggregate

Table 5. Concrete mix proportions

Symbol	Compressive strength, f'_c: MPa				
	Specimen 1	Specimen 2	Specimen 3	Mean	Standard deviation
MSD	23·6	28·7	34·4	28·9	5·4
MWC80W	8·2	10·0	11·2	9·8	1·5
MWC80C	8·0	9·6	11·5	9·7	1·8
MSA35FA	17·5	23·8	29·2	23·5	5·9
MSA50FA	25·3	31·8	37·4	31·5	6·1
MSA35CA	24·6	31·7	37·6	31·3	6·5
MSA50CA	22·7	27·3	31·6	27·2	4·5
MWC35W	41·1	50·6	57·4	49·7	8·2
MWC65W	12·4	14·9	17·4	14·9	2·5
MWC35C	45·4	56·3	63·9	55·2	9·3
MWC65C	13·4	15·7	19·2	16·1	2·9
MWC45W	31·5	35·3	44·7	37·2	6·8
MWC55W	20·4	24·0	30·0	24·8	4·8
MWC45C	30·0	38·1	43·9	37·3	7·0
MWC55C	20·0	23·6	29·6	24·4	4·9
MSA39FA	22·4	26·6	32·7	27·2	5·2
MSA45FA	24·7	29·8	36·1	30·2	5·7
MSA39CA	24·0	30·1	35·6	29·9	5·8
MSA45CA	23·6	27·4	33·1	28·0	4·8

Table 6. Compressive strength test results

Symbol	Elastic modulus, *E*: MPa				
	Specimen 1	Specimen 2	Specimen 3	Mean	Standard deviation
MSD	22 700	24 985	27 344	25 010	2322
MWC80W	8450	12 017	14 084	11 517	2850
MWC80C	17 558	19 744	21 450	19 584	1951
MSA35FA	22 025	23 580	26 011	23 872	2009
MSA50FA	25 685	27 958	30 452	28 032	2384
MSA35CA	24 542	26 392	29 142	26 692	2315
MSA50CA	20 574	24 366	26 292	23 744	2909
MWC35W	29 420	30 919	33 068	31 136	1834
MWC65W	16 483	19 270	21 895	19 216	2706
MWC35C	29 359	30 558	33 457	31 125	2107
MWC65C	19 080	21 428	22 849	21 119	1903
MWC45W	25 279	28 866	29 345	27 830	2222
MWC55W	20 397	23 589	25 578	23 188	2614
MWC45C	24 437	26 750	28 469	26 552	2023
MWC55C	21 286	24 445	25 027	23 586	2013
MSA39FA	22 140	24 108	26 286	24 178	2074
MSA45FA	24 448	25 447	28 579	26 158	2155
MSA39CA	24 213	24 818	28 422	25 818	2276
MSA45CA	21 919	23 994	27 380	24 431	2757

Table 7. Elastic modulus test results

Performance evaluation methodology

Satisfaction curve developments

The relationships between each concrete mixture parameter (i.e. water content w, cement content c, water-to-cement w/c ratio, fine aggregate content fa and coarse aggregate content ca) and the concrete compressive strength performance requirement of 25 MPa is represented by the one-parameter satisfaction curve developed using the Bayesian method as presented previously (Kim *et al.*, 2009).

For example, the mix proportions considered in this design example are for a 25 MPa compressive strength standard based on the satisfaction curve for the w/c parameter. The results of the satisfaction curve for the w/c parameter are shown in Figure 6(a). Applying the same analysis to other parameters, the compressive strength satisfaction curves for the w, c, fa and ca parameters are developed and shown in Figures 6(b), 6(c), 6(d) and 6(e) respectively.

Similarly, the elastic modulus results tabulated in Table 7 are used to develop the elastic modulus satisfaction curves for the w/c, w, c, fa and ca parameters, as shown in Figure 7.

Satisfaction curve combination using the importance factor and the goodness value

Importance factor

As explained previously, importance factors are determined on the basis of past knowledge about the parameter's degree of importance to the mixture. The importance factor of each concrete mixture parameter used in Equation 3 is assumed in Table 8. It is important to note that these values are used only hypothetically for concrete satisfying normal strength criteria in this example and can be changed for other criteria and cases. In addition, the total sum of all of the importance factors used for a batch cannot exceed 1·0. This is to prevent or limit haphazard usage of the importance factor since the selection of importance factors is subject to bias.

Goodness value

To combine the parameters' satisfaction curves, the parameters must be expressed as common values. Using Equation 2 with the minimum and maximum values of the concrete material parameters described in Table 9, the material parameters can be converted to goodness values in a range from 0 to 1. For example, the goodness value of the w/c parameter is calculated as (w/c − 0·35)/(0·8 − 0·35), where w/c has a value in the range 0·35 to 0·8.

Combination of satisfaction curves

The satisfaction curve combination process is performed to express multiple satisfaction curve characteristics as a single unique satisfaction curve on the basis of a threshold criterion. As defined previously, the goodness values of standard mixture parameters are chosen as the reference values, equivalent to a goodness value of 0·5. Therefore, the satisfaction curve of each parameter will be shifted according to a goodness value of 0·5 using the compatible values shown in Table 9. For example, the w/c parameter of the standard mix proportion is 0·5. The shifted compatible value of the goodness value of the w/c parameter is thus 0·5 − (0·5 − 0·35)/(0·8 − 0·35) = 0·1667. As for the positive trend conversion, a conversion is required for the w/c and w parameters when a strength criterion is applied. For example, an increase in the w/c and w parameters will result in decreased strength; hence, in order to combine the material parameter satisfaction curves into a single overall satisfaction curve for each level, the trend is converted into a 'positive' trend by changing the G_i value to $(1 - G_i)$ while strictly maintaining the compatible probability P%. Consequently, the *x*-axis values of the strength and elastic modulus satisfaction curves shown in Figure 6 and Figure 7 can be recalculated to produce those in Figure 8 and Figure 9, respectively.

Using the satisfaction curves of the concrete mix parameters, the satisfaction probabilities P_i are obtained for the goodness value G_i. For example, using Figure 8(a), when the w/c ratio value is 0·45, the relative goodness value G_i is 0·39 and the probability of satisfaction for a 25 MPa compressive strength P_i is 45%. The combination process is then implemented by applying Equation 3. The final combined satisfaction curves, which represent the overall trend for a concrete mixture satisfying the compressive strength and elastic modulus criteria of 25 MPa and 25 GPa, are shown in Figure 10.

In this example, the selected performance criteria refer to the compressive strength and elastic modulus. Hence, the combined compressive strength and elastic modulus satisfaction curves are combined to obtain a level 2 category of safety for the concrete mix proportion shown in Figure 11. This process of combining the compressive strength and elastic modulus satisfaction curves is performed using Equation 3, considering the importance factors of the criteria assumed in Table 10. The importance factor of compressive strength increases, whereas the importance factor of the elastic modulus decreases in the importance factor combinations given in safety_1, safety_2 and safety_3. The probability of satisfaction as calculated using these importance factor combinations shows that the probability of safety satisfaction increases. Obviously, if the design engineer has to consider other performance criteria such as slump, flow, water penetration, creep and shrinkage, other important factor combinations can be used to combine the satisfaction curves. Once all of these one-parameter satisfaction curves are combined for a given level, the process can be repeated at the next higher level until the satisfaction curves for constructability, durability and damageability are developed, providing the overall satisfaction trends for a given mix proportion. However, the selected performance criteria in this simple example include only compressive strength and elastic modulus, which were chosen to describe the method as clearly as possible.

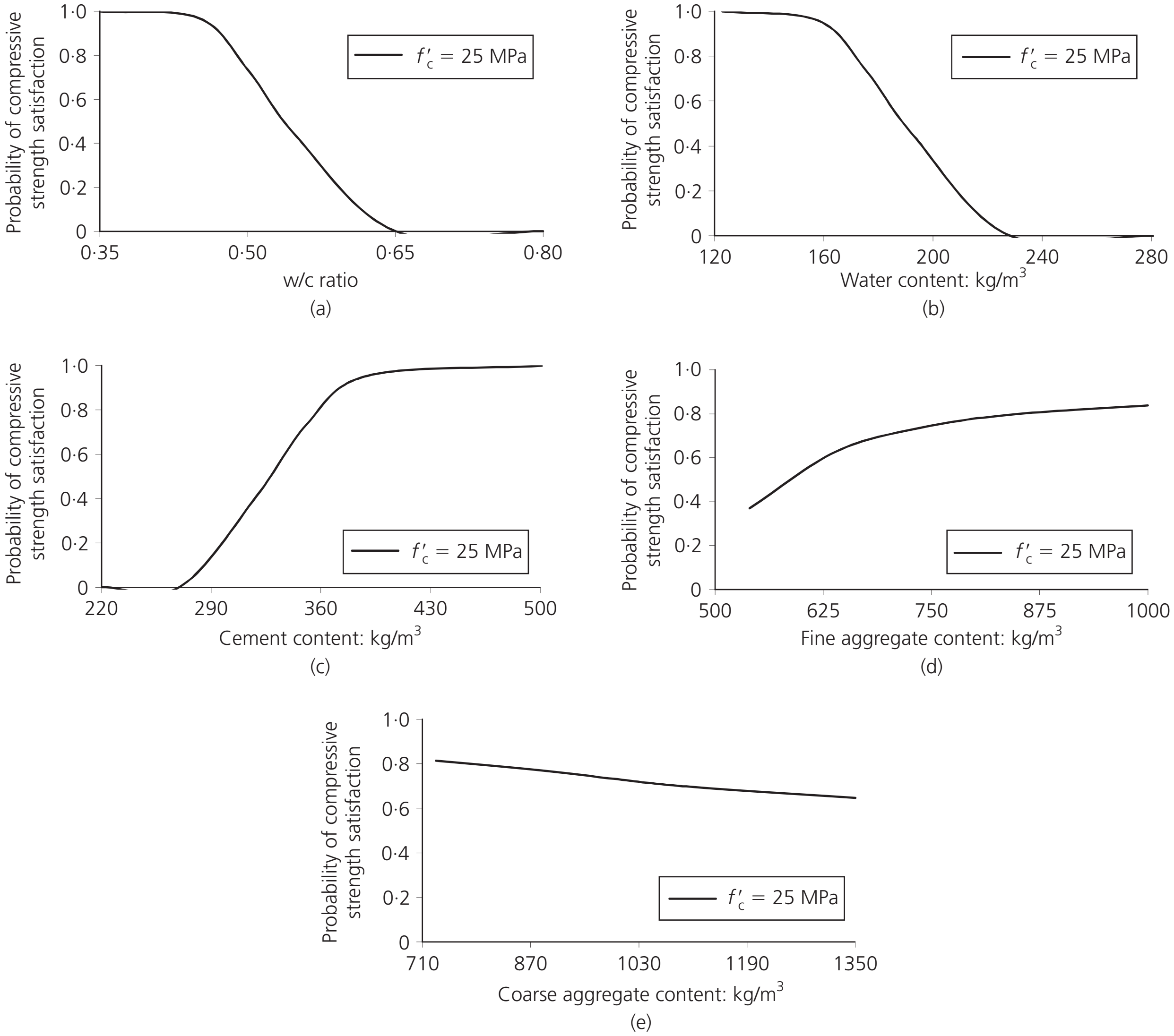


Figure 6. Compressive strength satisfaction curve of: (a) w/c ratio; (b) water content; (c) cement content; (d) fine aggregate content; and (e) coarse aggregate content

Mix proportion design verification and modification

From original request to actual evaluation comparison

The design standard mixture proportion is used to evaluate the material satisfaction probabilities, which must satisfy the required probability satisfaction P%. In this example, the goodness values of all of the parameters in an example standard mixture are set to 0·5, which can be considered a 'reference point'. The respective probability satisfactions for a 25 MPa compressive strength and a 25 GPa elastic modulus are 74% and 61%, respectively, obtained from Figure 10. The compressive strength and elastic modulus satisfaction probabilities are greater and less, respectively, than the required probability of 65%. The difference in the satisfaction probability is expressed as follows

$$\text{Probability difference} = \left(\frac{P - P_{\text{PBMD}}}{P}\right)100\% \tag{4}$$

where P is the required criterion satisfaction probability and P_{PBMD} is the required criterion satisfaction probability using the PBMD method for a material parameter.

Using Equation 4, the probability differences in compressive strength and the elastic modulus are 13·8% and −6·1% respectively.

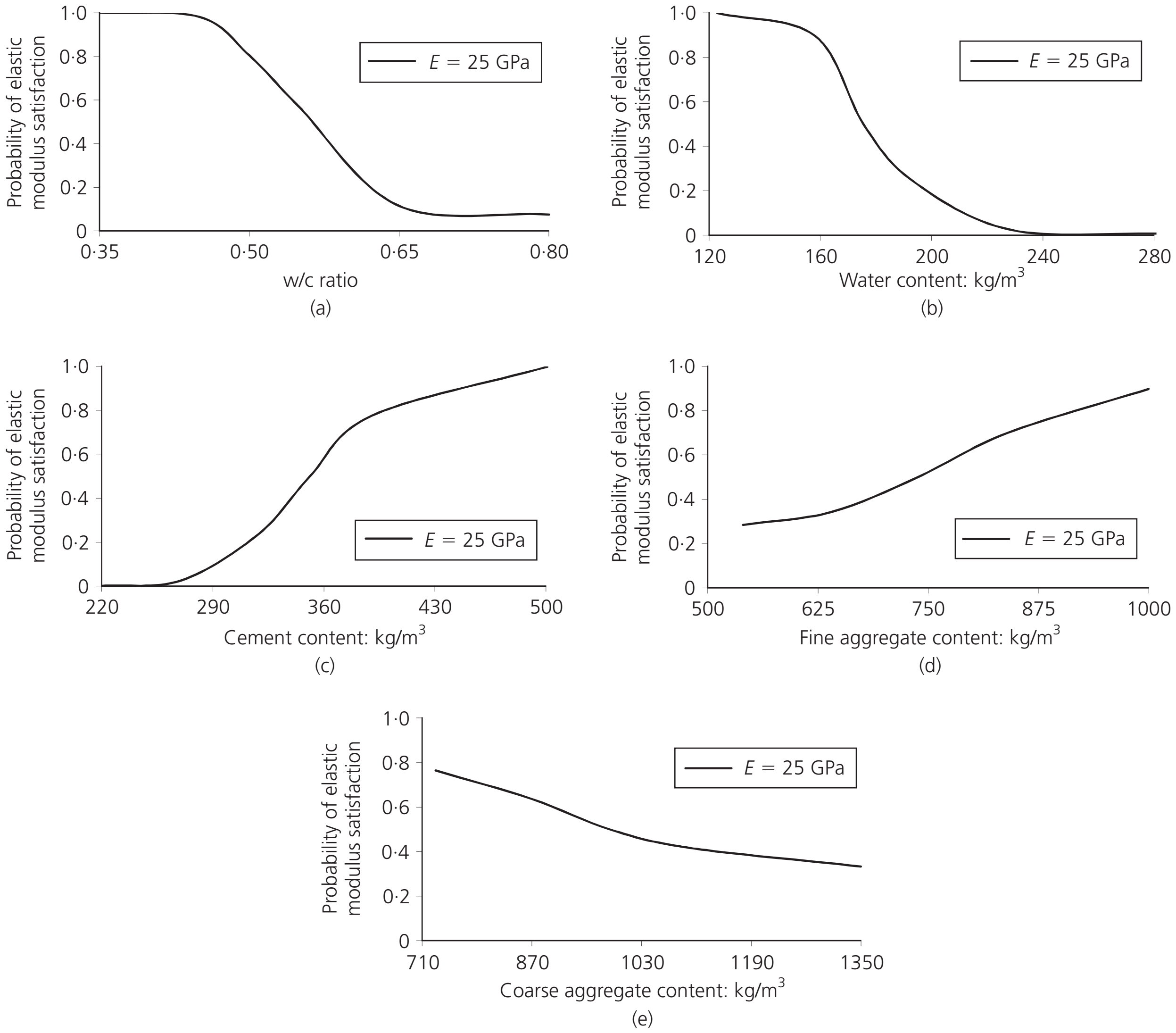


Figure 7. Elastic modulus satisfaction curve of: (a) w/c ratio; (b) water content; (c) cement content; (d) fine aggregate content; and (e) coarse aggregate content

Parameter	Importance factor	
	Compressive strength	Elastic modulus
w/c ratio	0·50	0·35
Water content	0·10	0·25
Cement content	0·30	0·30
Fine aggregate content	0·05	0·05
Coarse aggregate content	0·05	0·05
. . .	0·00	0·00
Total	1·00	1·00

Table 8. The importance factors for the concrete parameters

Concrete material parameter	Min. value of parameter	Max. value of parameter	Shifted compatible value
w/c ratio	0·35	0·80	0·1667
Water content: kg/m^3	122·9	280·9	0·1667
Cement content: kg/m^3	219·5	501·6	0·0335
Fine aggregate content: kg/m^3	540	1000	0·0713
Coarse aggregate content: kg/m^3	730	1350	0·0874

Table 9. The concrete material parameter values

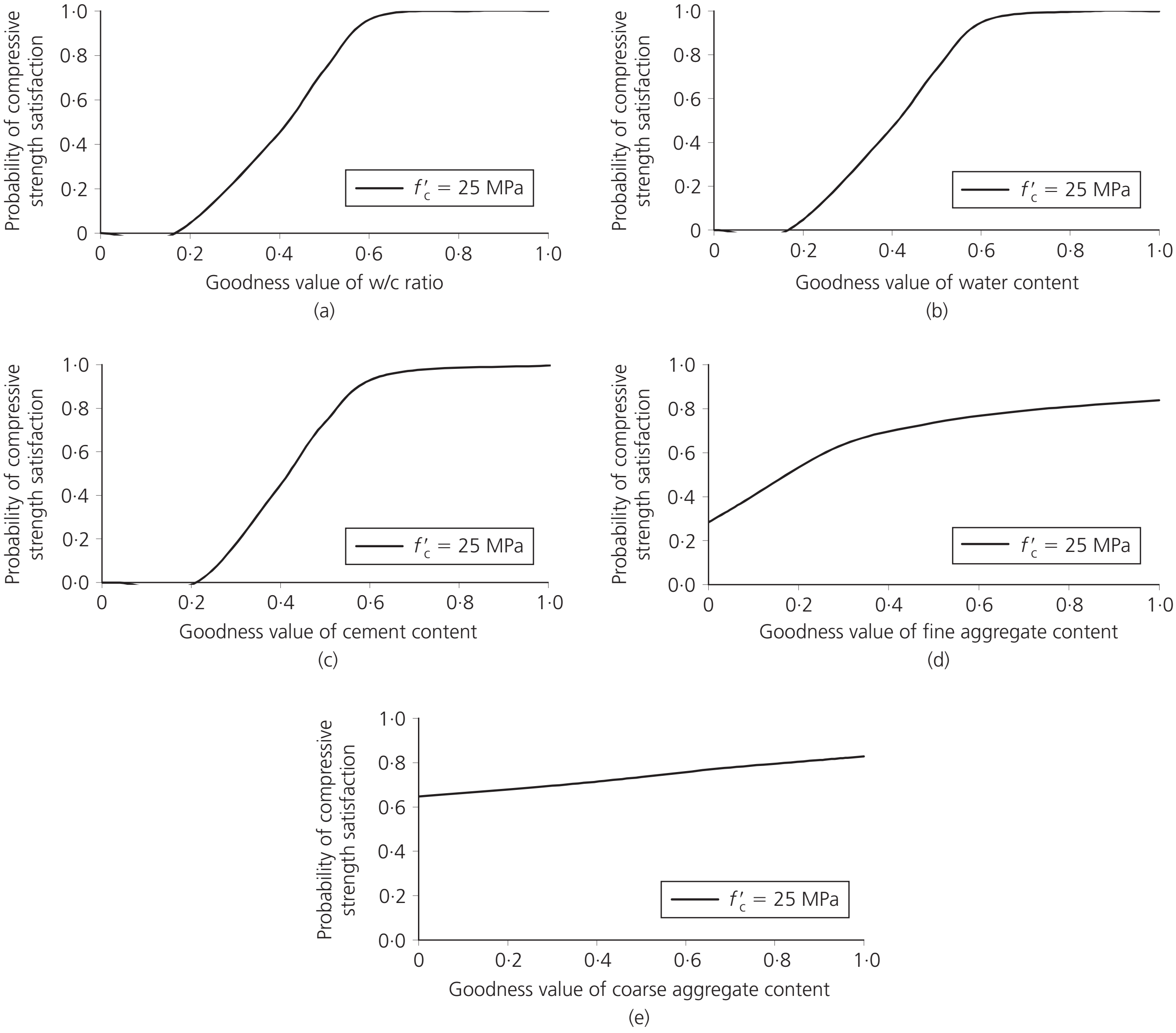
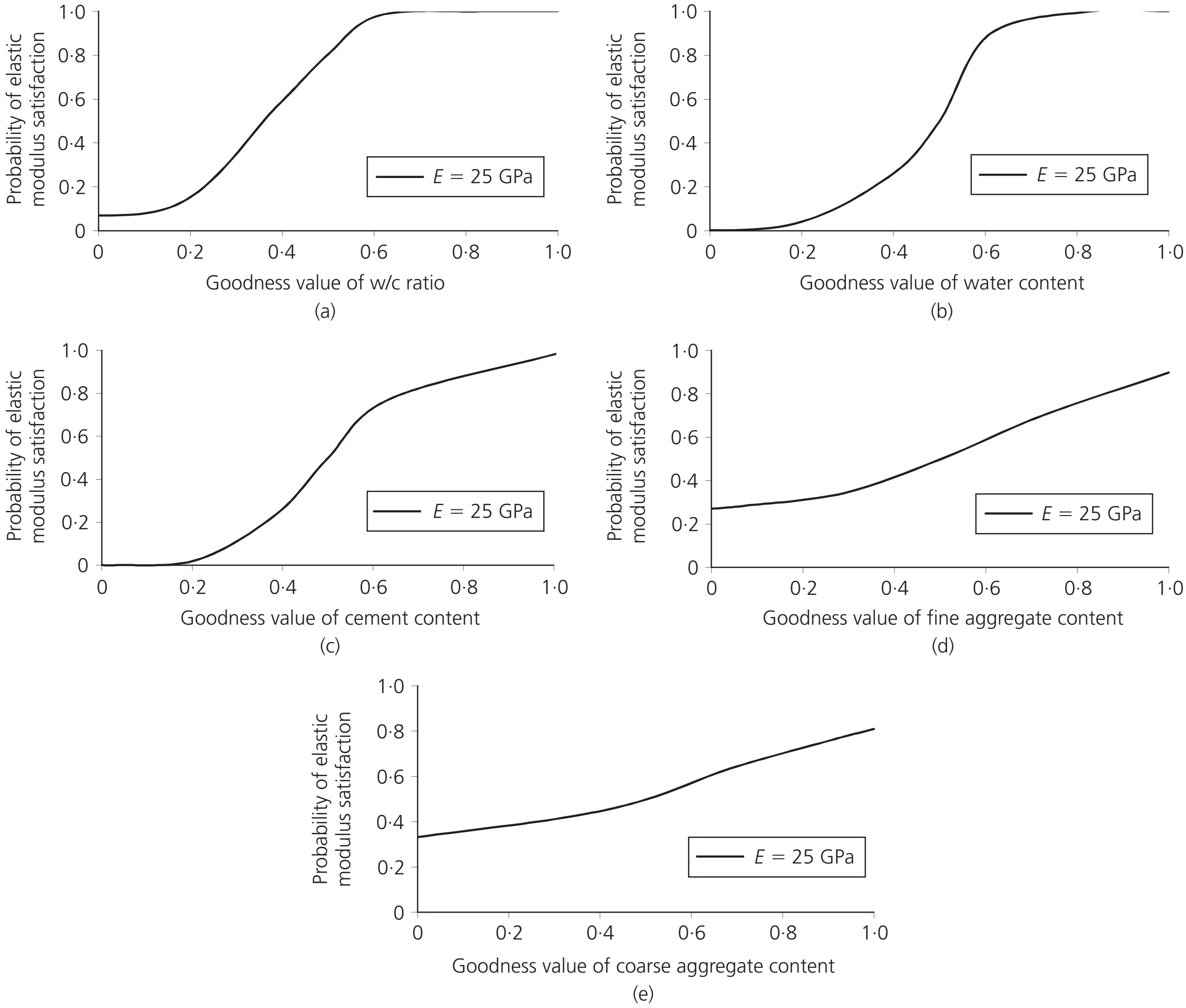
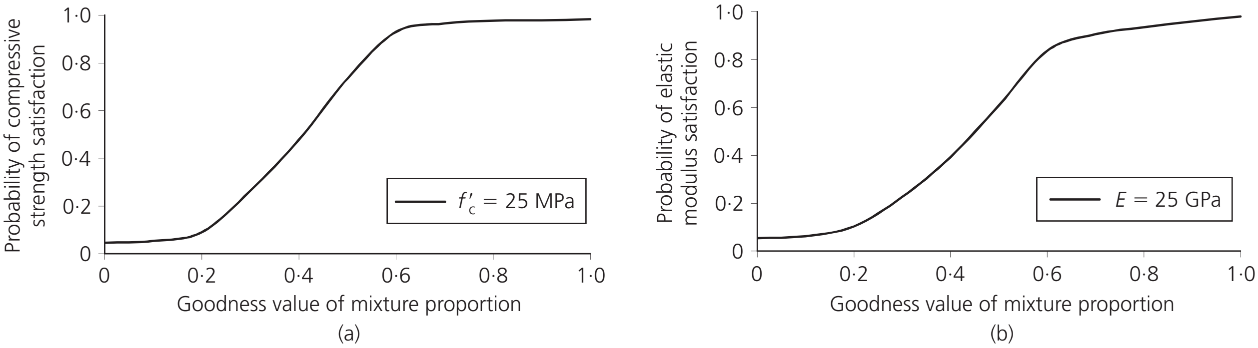


Figure 8. Compressive strength satisfaction curve of the goodness values of: (a) w/c ratio; (b) water content; (c) cement content; (d) fine aggregate content; and (e) coarse aggregate content



(a) (b) (c) (d) (e)

Figure 9. Elastic modulus satisfaction curve of the goodness values of: (a) w/c ratio; (b) water content; (c) cement content; (d) fine aggregate content; and (e) coarse aggregate content



(a) (b)

Figure 10. Combined (a) compressive strength and (b) elastic modulus satisfaction curve for the concrete mix proportion

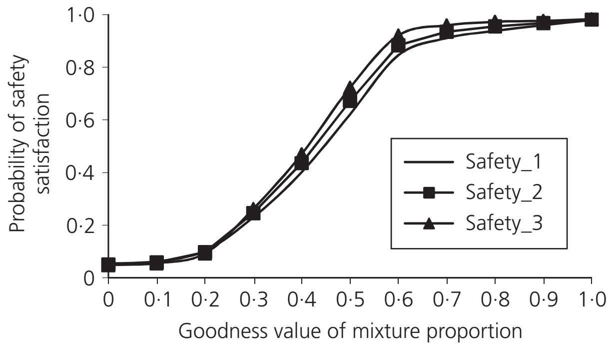


Figure 11. Combined safety satisfaction curves for the concrete mix proportion

Parameter	Importance factor		
	Safety_1	Safety_2	Safety_3
Compressive strength	0·2	0·5	0·8
Elastic modulus	0·8	0·5	0·2
. . .	0·0	0·0	0·0
Total	1·0	1·0	1·0

Table 10. The importance factors for the safety satisfaction curve

Concrete mix proportion modification

Generally, an initially selected standard mix proportion (MSD) cannot satisfy all of the required design target criteria in a single trial. This means that a mix proportion design can only satisfy some of the selected criteria. Figure 10 shows that the probabilities of satisfying the criteria increase as the goodness values increase. Therefore, if the mix proportion must satisfy the required compressive strength and elastic modulus satisfaction probabilities, the mix proportion needs to be modified so that the range of the goodness values in the mix proportion is higher. In this case, the satisfaction probability of the compressive strength will be satisfied, but the satisfaction probability of the elastic modulus deviates slightly from the required value. Since, it is more important to satisfy the main criterion among all of the selected ones, in this example, the main criterion that a mix proportion must satisfy is concrete compressive strength, whereas the elastic modulus criterion can be considered less important. Therefore, even though the satisfaction probability of the elastic modulus is still less than the required value (6·1%), the concrete mix proportion does not need to be modified. However, in order to illustrate the modification details of the concrete mix proportion to satisfy the required value, this concrete mix proportion will be modified.

In order to modify the mix proportion, a careful understanding of the importance factor is essential, where the importance factor of each parameter for a specific concrete criterion greatly influences the performance of the mix proportion in the PBMD. Based on the importance factors of the concrete parameters for the compression strength criterion shown in Table 8, the main parameters of the mix proportion are w/c and c. Therefore, these two parameters are modified, and other parameters are kept constant. The w/c and c parameter values of 0·5 and 351·13 (kg/m^3) will be replaced by 0·49 and 358·3 (kg/m^3), respectively. The goodness values of w/c and c are $(0{\cdot}49 - 0{\cdot}35)/(0{\cdot}8 - 0{\cdot}35) + 0{\cdot}1667 = 0{\cdot}48$ and $(358{\cdot}3 - 219{\cdot}5)/(501{\cdot}6-219{\cdot}5) + 0{\cdot}0335 = 0{\cdot}53$, respectively. Because the trend of the satisfaction curve of w/c needs to be converted to a ‘positive’ trend, the final goodness value of w/c is $1 - 0{\cdot}48 = 0{\cdot}52$. The modified mix proportion’s parameter values and the goodness values of concrete standard parameters are listed in Table 11.

The goodness values of the modified concrete standard mix proportion for compressive strength and elastic modulus can be combined in the following equation

$$G = G_i \alpha_i \tag{5}$$

where G_i is the goodness value of the material parameters and α_i represents the importance factors of the material parameters in the mixture design for a specified criterion.

The satisfaction curves developed as goodness values on the x-axis and the compatible satisfaction probabilities on the y-axis for the modified concrete standard mix proportion’s compressive strength and elastic modulus are shown in Figures 10(a) and 10(b), respectively, and are listed in Table 12. The final 77% and 66% satisfaction probabilities for compressive strength and elastic modulus, respectively, obtained from the modified concrete standard mix proportion are greater than the required probability, indicating that the mix proportion design satisfies the required satisfaction probabilities. Hence, the modified concrete mix proportion can be considered an acceptable design.

Client requests verification for final concrete mix proportion design

In this step of the design process, the satisfaction trend has to be verified by a client. Depending on the concrete’s performance

Parameter	Standard mix proportion	Modified mix proportion	Parameter’s goodness Value
w/c	0·50	0·49	0·52
w: kg/m^3	175·57	175·57	0·50
c: kg/m^3	351·13	358·30	0·53
fa: kg/m^3	737·21	737·21	0·50
ca: kg/m^3	985·79	985·79	0·50

Table 11. Modified concrete standard mix proportion

Parameter	Parameter's goodness value	Importance factor		Goodness value of modified mix		Probability of satisfaction: %	
		f'_c	E	f'_c	E	f'_c	E
w/c	0·52	0·50	0·35	0·519	0·516	77	66
w	0·50	0·10	0·25				
c	0·53	0·30	0·30				
fa	0·50	0·05	0·05				
ca	0·50	0·05	0·05				

Table 12. Goodness value and satisfaction probability of the modified concrete standard mix proportion

class, the importance factor values of these criteria are assumed, and a compatible safety satisfaction curve trend is obtained. In this example, the ordinary safety requirement has to be satisfied for the mix design, and therefore compressive strength is considered more important than the elastic modulus. Hence, the importance factor of safety_3 among the selections listed in Table 10 is used. The safety satisfaction curve trends shown in Figure 11 are sufficient for use in construction. The safety satisfaction curves increase linearly in the goodness value range from 0·2 to 0·6 and remain mostly horizontal in the ranges from 0·0 to 0·2 (unsatisfactory) and from 0·6 to 1·0 (satisfactory). The trend of the satisfaction curves indicates that a change in the goodness value of a material parameter does not cause a drastic change in the satisfaction probability. As mentioned previously, if the satisfaction curve trend has a large unsatisfied range or changes drastically in a narrow range of goodness values, one can conclude that, because of the satisfaction curve trend, the concrete is not sufficiently safe for use in construction.

Verification and discussion

The performance of the modified standard design mixture (MSDM) proportion is verified using the compressive strength and elastic modulus experimental results. The experimental results are shown in Table 13. The probabilities of satisfaction for a 25 MPa compressive strength and a 25 GPa elastic modulus criterion are calculated on the basis of these compression test data using the method presented in ACI 214R-02. With respect to ACI compressive strength distribution probability, the maximum range was approximately 95·45%.Therefore, we chose a 5% difference accuracy based on 100 − 95·45, which amounted to an approximately 5% accuracy limit. Consequently, when the result comparison difference was less than 5%, it was assumed that the results were within an allowable range.

The probabilities of exceeding the expected 25 MPa compression strength and 25 GPa elastic modulus criteria calculated by ACI 214R-02 are 74% and 68·6% respectively. The differences between ACI and the PBMD method for compression strength and elastic modulus are 4·1% and 3·9% respectively, and are within the allowable range of 5%.

MSDM	Compressive strength: MPa	Elastic modulus: MPa
1	22·1	22 445
2	29·3	26 865
3	31·2	28 694
4	23·1	23 398
5	28·9	27 757
6	30·5	29 035
Mean	27·5	26 349
Standard deviation	3·91	2773

Table 13. Supplementary tests for MSDM

In conclusion, a concrete mix proportion with design requirements that set the compressive strength and the elastic modulus at 25 MPa and 25 GPa, respectively, can be classified into the 'moderate performance class', for which the required probability of exceeding this criterion is approximately 65%. The final modified concrete mix proportion (w/c = 0·49, w = 175·57 kg/m^3, c = 358·30 kg/m^3, fa = 737·21 kg/m^3, ca = 985·79 kg/m^3) resulted in satisfaction probabilities of 77% and 66% for the compressive strength and the elastic modulus, respectively. The experimental results obtained from this mix proportion are 74% and 68·6% with mean compressive strength and elastic modulus values of 27·5 MPa and 26·3 GPa, respectively, as calculated by ACI 214R-02. The similarity of the results shows that the modified concrete mix proportion satisfies the required performance criteria as well as the satisfaction probabilities. Furthermore, the analytical results obtained from the one-parameter method using the PBMD scheme are shown to be sufficiently reliable to be used in a concrete mix proportion design.

Conclusions

The following conclusions can be drawn from the findings of this paper.

(*a*) A step-by-step example of a mix proportion design which satisfies the given usage requirements is provided using the proposed PBMD procedure. It offers general and detailed examples of the PBMD procedure used to derive the probability of satisfying concrete performance criteria at each material parameter level.

(*b*) The goodness value concept is proposed in the PBMD procedure so that different parameters can be converted into a common parameter.

(*c*) The importance factor concept for each parameter is introduced through the combination of the satisfaction curves and by modifying the concrete mix proportion design.

(*d*) The design example indicates that the proposed PBMD procedure can be implemented for designing a mix proportion and evaluating the probability of satisfying the concrete performance criteria with a given material parameter or mixture proportion.

(*e*) The difference between the analytical results and the results obtained using the method presented in ACI 214R-02 is less than 5%, which is within an acceptable range. Therefore, the proposed method can be used to design and predict the satisfaction probability of concrete performance criteria for a realistic range of material parameter values.

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