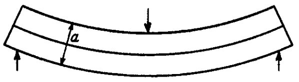
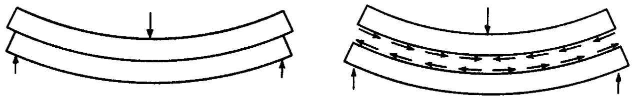
Chapter 6 전단강도 및 설계 (Shear Strength & Design)

6.1 탄성단면 해석 (Elastic Section Analysis)

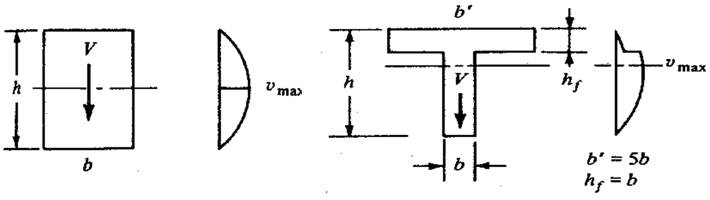


◉ Shear Stress for V only :



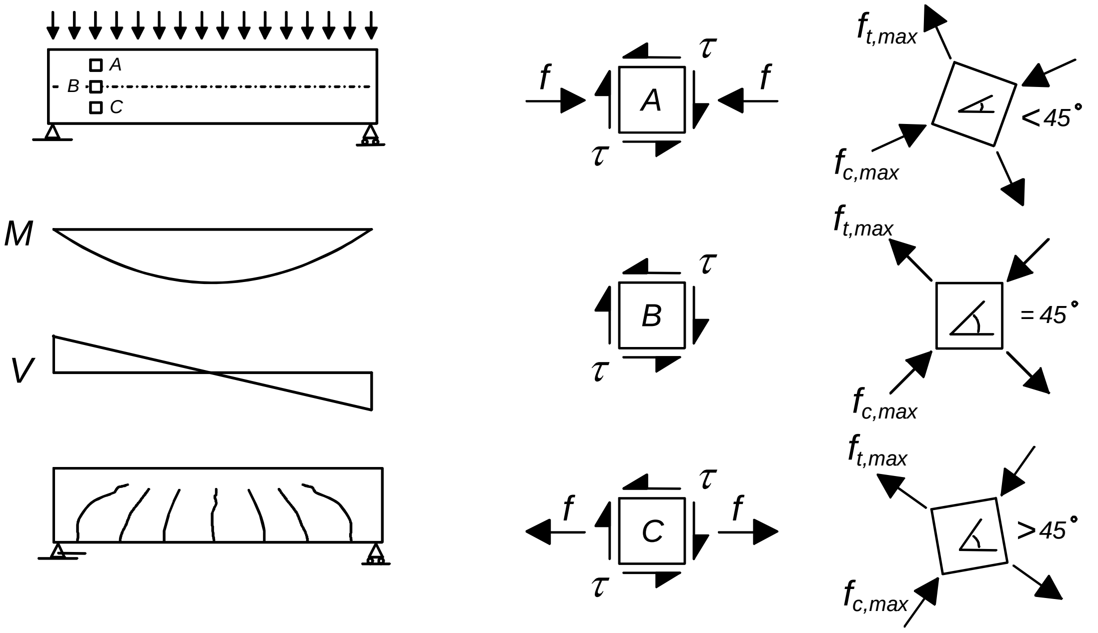
$$\tau = \frac{VQ}{Ib}$$

Q : 단면 1차 모멘트

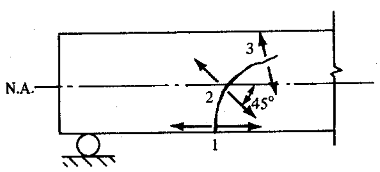


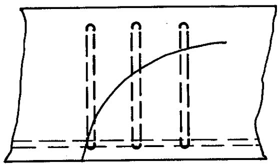
◉ Maximum Principle Stress for $V + M$:

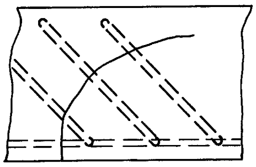
$$\begin{pmatrix} f_{t,\max} \\ f_{c,\max} \end{pmatrix} = \frac{1}{2}f \pm \sqrt{\left(\frac{1}{2}f\right)^2 + \tau^2} \qquad \tan 2\theta = -\frac{2\tau}{f}$$



- 전단철근 (Stirrup, 스터럽) : 인장응력방향으로 배근





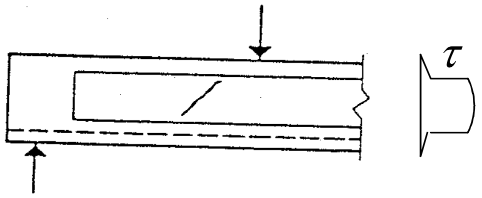


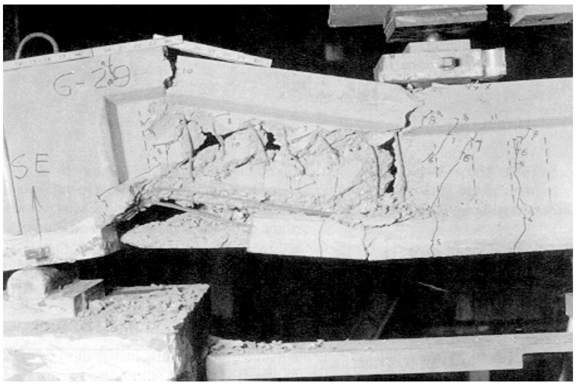
- 탄성해석은 균열을 고려하지 않은 균질재료(homogeneous material)에 대한 이론적 해석이므로 균열발생 후에는 거동을 예측할 수 없다.

6.2 스터럽이 없는 경우의 전단강도 (Shear Strength without Stirrup)

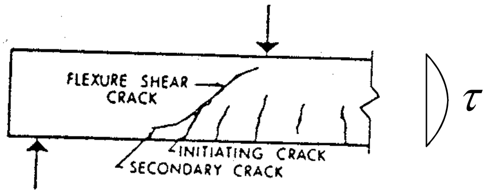
◉ Shear Cracks

(1) Web-Shear Crack : 주로 PSC



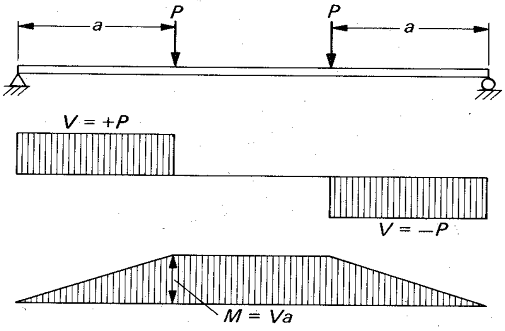


(2) Flexure-Shear Crack : RC, PSC



◉ Shear Behavior

- Depends on shear span, *a*.

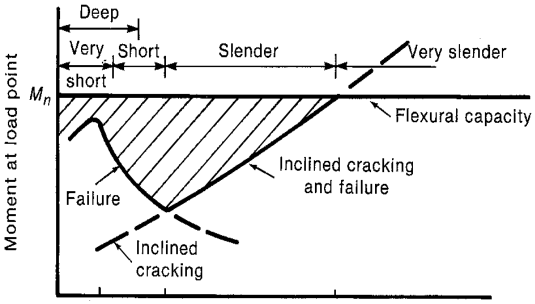


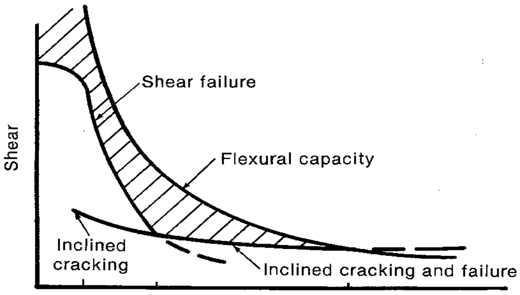
$a = \frac{M}{V}$: shear span (전단지간)

$\frac{a}{d}$: shear span-depth ratio

전단거동에 대한 중요 요소.

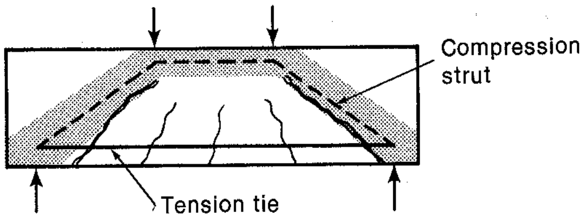
- Categories due to $\frac{a}{d}$ ratio

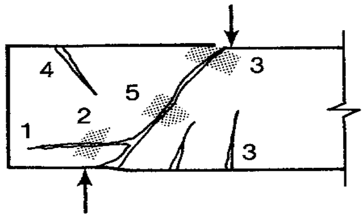




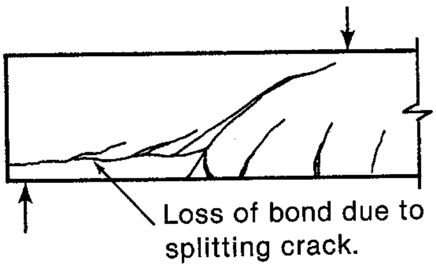
- Four Categories

(1) Deep Beam (깊은 보) : $\frac{a}{d} \le 1$

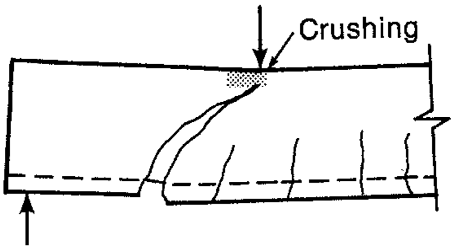




(2) Short Beam : $1 < \frac{a}{d} \le 2.5$



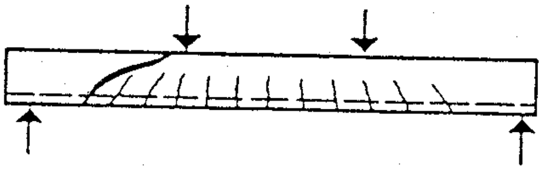
Shear Tension Failure



Shear Compression Failure

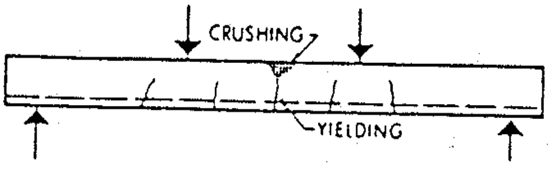
(3) Usual Beam : $2.5 < \frac{a}{d} \le 6$ (Diagonal Tension Failure)

· 수직휨균열이 먼저 발생한 후 사인장 균열이 발생하여 파괴에 이른다.

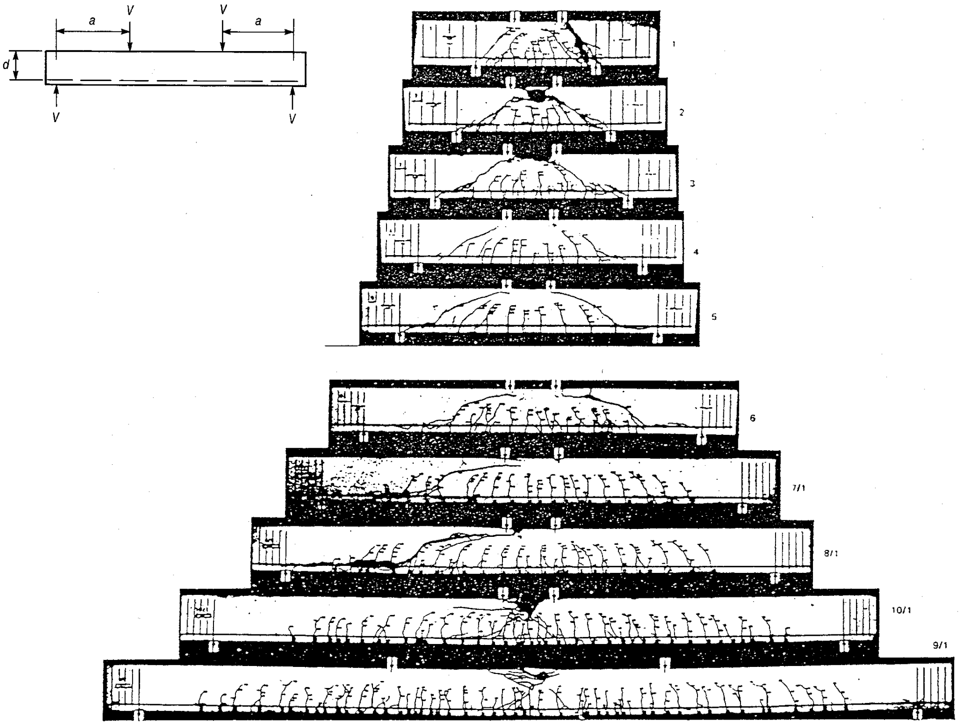


Diagonal Tension Failure

(4) Long Beam : $\frac{a}{d} > 6$ 주로 휨파괴



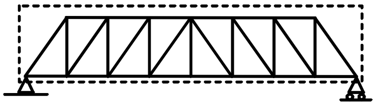
- Failure Pattern

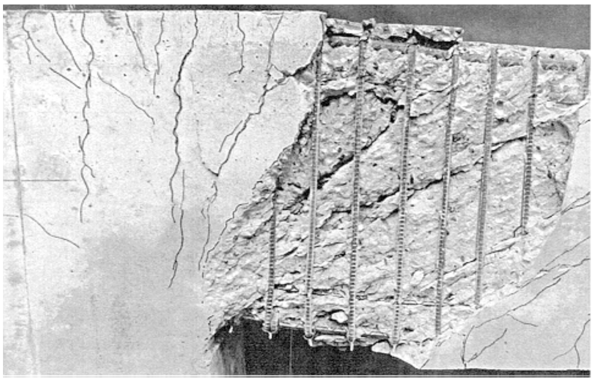


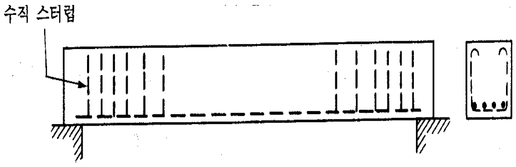
실험체	1	2	3	4	5	6	7/1	8/1	9/1	10/1
a/d	1.0	1.5	2.0	2.5	3.0	4.0	5.0	6.0	8.0	7.0

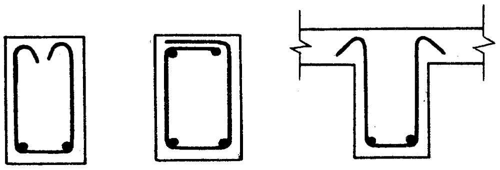
6.3 스터럽이 있는 경우의 전단강도 (Shear Strength with Stirrups)

◉ Function of Shear Reinforcement : *Truss Analogy*





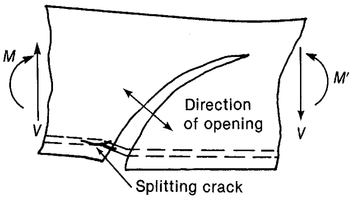




◉ Nominal Shear Strength : $V_n = V_c + V_s$ [콘설기 7.2.1(1), 도설기 4.4.6.1]

→ Contribution by *Concrete* & *Shear Reinforcement*

◉ Cracked Concrete Shear Strength : $V_c = V_{cz} + V_i + V_d$



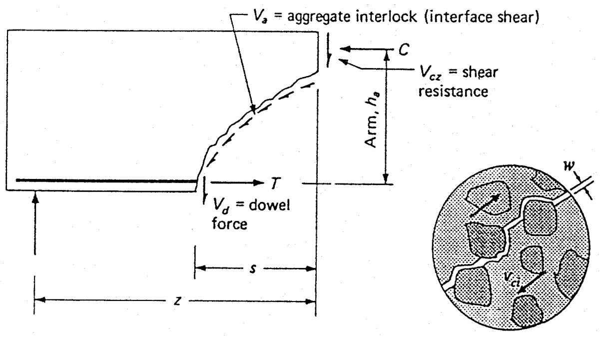
V_{cz} : Uncracked Concrete Shear Strength. 33~50%

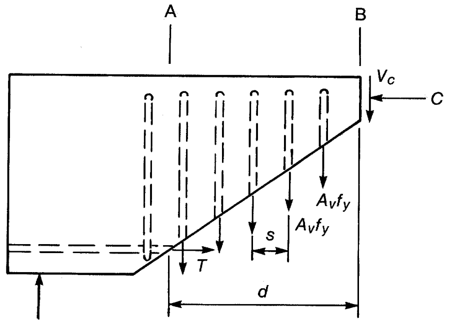
V_i : Aggregate Interlocking, 20~40%

V_d : Dowel Action, 15~20%

[콘설기 7.3.1(1)] 실용식 $V_c = \frac{1}{6}\sqrt{f_{ck}}\, b_w d$ [MPa, mm] : $\sqrt{f_{ck}} \le 8.4$ MPa

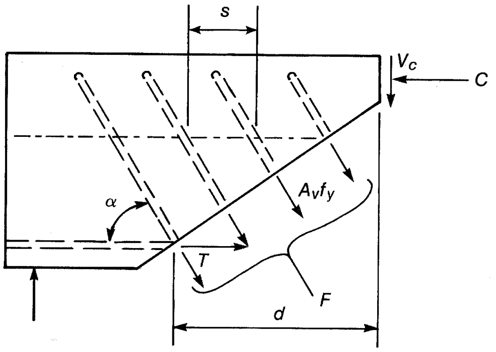
[콘설기 7.3.1(2)] 엄밀식 $V_c = \left(0.16\sqrt{f_{ck}} + 17.6\frac{\rho_w V_u d}{M_u}\right) b_w d \le 0.3\sqrt{f_{ck}}\, b_w d$

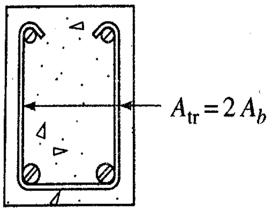


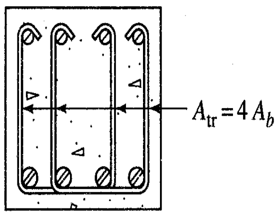


◉ Shear Strength by stirrup : $V_s = \sum A_v f_y$

사인장 균열각도를 45°로 가정하고 균열 내의 전단철근량(n) 결정



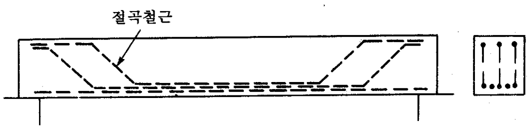




▪ 수직 Stirrup : $V_s = n\, A_v f_y = \frac{d}{s} A_v f_y = \frac{A_v f_y d}{s}$ [콘설기 7.4.4(2)]

▪ 경사 Stirrup : $V_s = n\, A_v f_y = \frac{A_v f_y (\sin\alpha + \cos\alpha)\, d}{s}$ [콘설기 7.4.4(4)]

◉ Design Shear Strength & Safety



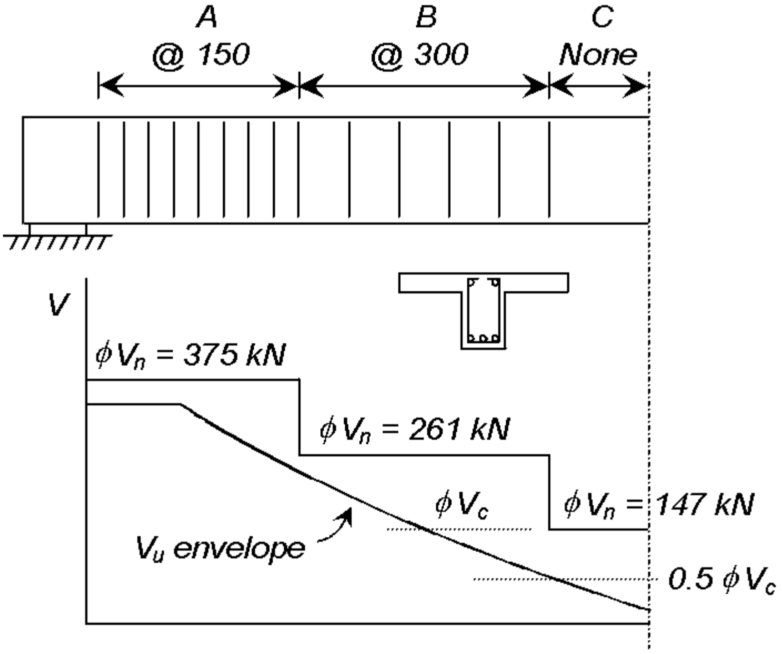
$\phi V_n = \phi V_c + \phi V_s$

$\phi V_n \ge V_u$ $\phi_v = 0.75$ [콘설기 3.3.3(2)] $\phi_v = 0.8$ [도설기 4.4.2.2]

◉ Example 6-1 : 콘크리트구조설계기준(2007)에 따른 공칭전단강도 V_n와 설계전단강도 ϕV_n

b_w = 400 mm d = 600 mm f_{ck} = 24 MPa

SD300, D13 U-Stirrup → $A_v = 2A_b = 253\ \text{mm}^2$



[실용식] $V_c = \dfrac{1}{6}\sqrt{f_{ck}}\,b_w d = \dfrac{1}{6}\sqrt{24}\,(400)(600)/1000 = 196\ \text{kN}$

[구간 A] For s = 150 mm, $V_s = \dfrac{A_v f_y d}{s} = \dfrac{253\,(300)(600)}{150\,(1000)} = 304\ \text{kN}$

$V_n = V_c + V_s = 196 + 304 = 500\ \text{kN}$

$\phi V_n = 0.75\,(500) = 375\ \text{kN}$

[구간 B] For s = 300 mm, $V_s = \dfrac{A_v f_y d}{s} = \dfrac{253\,(300)(600)}{300\,(1000)} = 152\ \text{kN}$

$V_n = V_c + V_s = 196 + 152 = 348\ \text{kN}$

$\phi V_n = 0.75\,(348) = 261\ \text{kN}$

[구간 C] $V_n = V_c = 196\ \text{kN}$

$\phi V_n = 0.75\,(196) = 147\ \text{kN}$

6.4 전단설계 (Design for Shear)

◉ Design Objectives

$\phi V_n \geq V_u$ $\phi V_n = \phi V_c + \phi V_s$ [콘설기 7.2.1(1), 도설기 4.4.6.1]

$\phi_v = 0.75$ [콘설기 3.3.3(2)] $\phi_v = 0.8$ [도설기 4.4.2.2]

◉ Required Stirrup Spacing

$$req'd\ \phi V_s = V_u - \phi V_c \qquad (note)\ \phi V_s = \frac{\phi A_v f_y d}{s}$$

$$\rightarrow req'd\ \phi V_s = \frac{\phi A_v f_y d}{req'd\ s} \quad \rightarrow \quad req'd\ s = \frac{\phi A_v f_y d}{req'd\ \phi V_s}$$

※ 설계에서 일반적으로 전단철근의 크기(A_v)를 결정한 후 간격을 조정한다.

◉ Minimum and Maximum Shear Reinforcement

If A_v *is too little,* 사인장균열이 즉시 발생하여 취성으로 파괴.

If A_v *is too high,* 전단철근이 항복하기 전에 *shear-compression failure* 발생 → 취성 파괴

- 바람직한 파괴양상 : 사인장 균열 발생 후에도 전단철근이 항복할 때까지 전단철근과 콘크리트 압축부가 전단력에 저항하여야 한다. → 최소 및 최대 전단철근량 결정

▣ 콘크리트구조설계기준 7.4 :

[콘설기 7.4.1(3)] $f_y \leq 400\ MPa$

[콘설기 7.4.3(3)] $A_{v,\min} = 0.0625\sqrt{f_{ck}}\,\frac{b_w s}{f_y} \geq 0.35\,\frac{b_w s}{f_y}$

$$\rightarrow \max\ s = \frac{A_v f_y}{0.0625\sqrt{f_{ck}}\,b_w} \text{ 와 } \frac{A_v f_y}{0.35\,b_w} \text{ 중 작은 값}$$

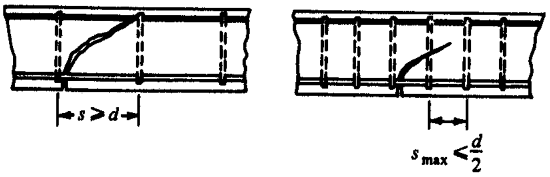
[콘설기 7.4.4(9)] $\max\ V_s = \frac{2}{3}\sqrt{f_{ck}}\,b_w d$

[콘설기 7.4.3(1)] $V_u > \frac{1}{2}\phi V_c$ 인 구간에 최소전단철근 배근.

예외 : 슬래브, 기초판, 장선구조, 전체 높이가 250 mm
이하인 보, 플랜지가 두꺼운 I형보와 T형보

◉ Maximum Stirrup Spacing (최대간격)

45° 사인장균열 내에 최소한 1개의 stirrup이 배근되도록 결정하였다.



[기준 7.4.2(1)] ① max $s = \dfrac{d}{2}$ or 600 mm

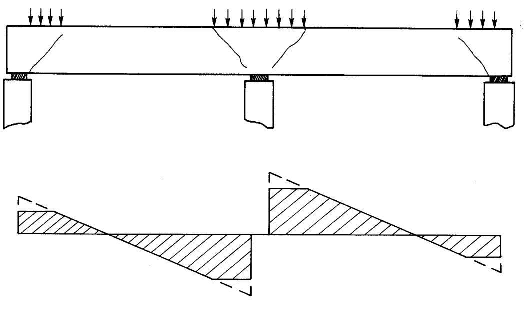
② $V_s > \dfrac{\sqrt{f_{ck}}}{3}\, b_w d$ 인 경우 → max $s = \dfrac{d}{4}$ or 300 mm

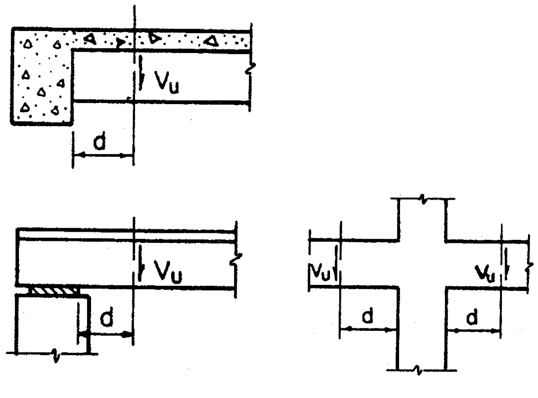
※ 1 more *"max s"* requirement by $A_{v,\min}$ [콘설기 7.4.3(3)]

→ max $s = \dfrac{A_v f_y}{0.0625\sqrt{f_{ck}}\, b_w}$ 와 $\dfrac{A_v f_y}{0.35\, b_w}$ 중 작은 값

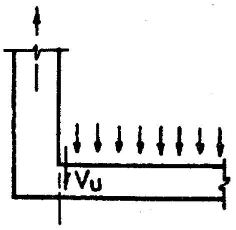
◉ Critical Section (위험단면) : 최대 전단력이 작용하는 위치

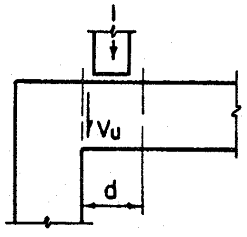
[콘설기 7.2.1(5)] 반력이 부재단부를 압축하고, 받침부 내면에서 *"d"* 거리 이내에 위치한 단면에 집중하중이 작용하지 않는 경우는 *"d"* 에서 구한 *"V_u"* 를 사용할 수 있다.

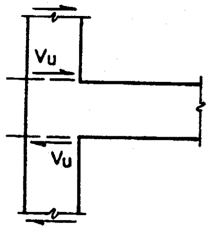


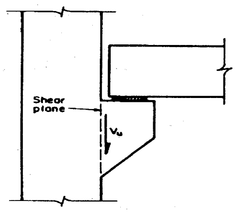


- 7.2.1(5) 이외의 경우, 위험단면의 위치는 받침부 내면.

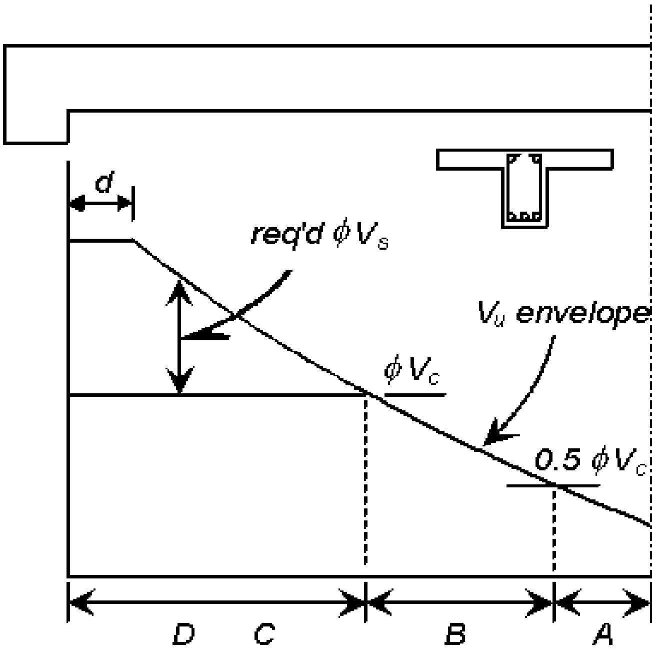








◉ Summary for Shear Design : $\phi V_n \geq V_u \qquad \phi V_n = \phi V_c + \phi V_s$



(1) A : $V_u \leq \dfrac{1}{2}\phi V_c$ → *No stirrups* (*note*) $V_c = \dfrac{1}{6}\sqrt{f_{ck}}\, b_w d$

(2) B : $\dfrac{1}{2}\phi V_c < V_u \leq \phi V_c$ [콘설기 7.4.3(1)] 슬래브와 기초판은 제외

$$\rightarrow \text{max } s = \frac{A_v f_y}{0.0625\sqrt{f_{ck}}\, b_w} \text{ or } \frac{A_v f_y}{0.35\, b_w} \text{ or } \frac{d}{2} \text{ or } 600 \text{ mm}$$

(3) C : $\phi V_c < V_u \leq \phi V_c + \phi\left(\dfrac{1}{3}\sqrt{f_{ck}}\right) b_w d$

$$\rightarrow req'd\ s = \frac{\phi A_v f_y d}{req'd\ \phi V_s} \qquad \text{max } s = \frac{d}{2} \text{ or } 600 \text{ mm}$$

(4) D : $\phi V_c + \phi\left(\dfrac{1}{3}\sqrt{f_{ck}}\right) b_w d < V_u \leq \phi V_c + \phi\left(\dfrac{2}{3}\sqrt{f_{ck}}\right) b_w d$

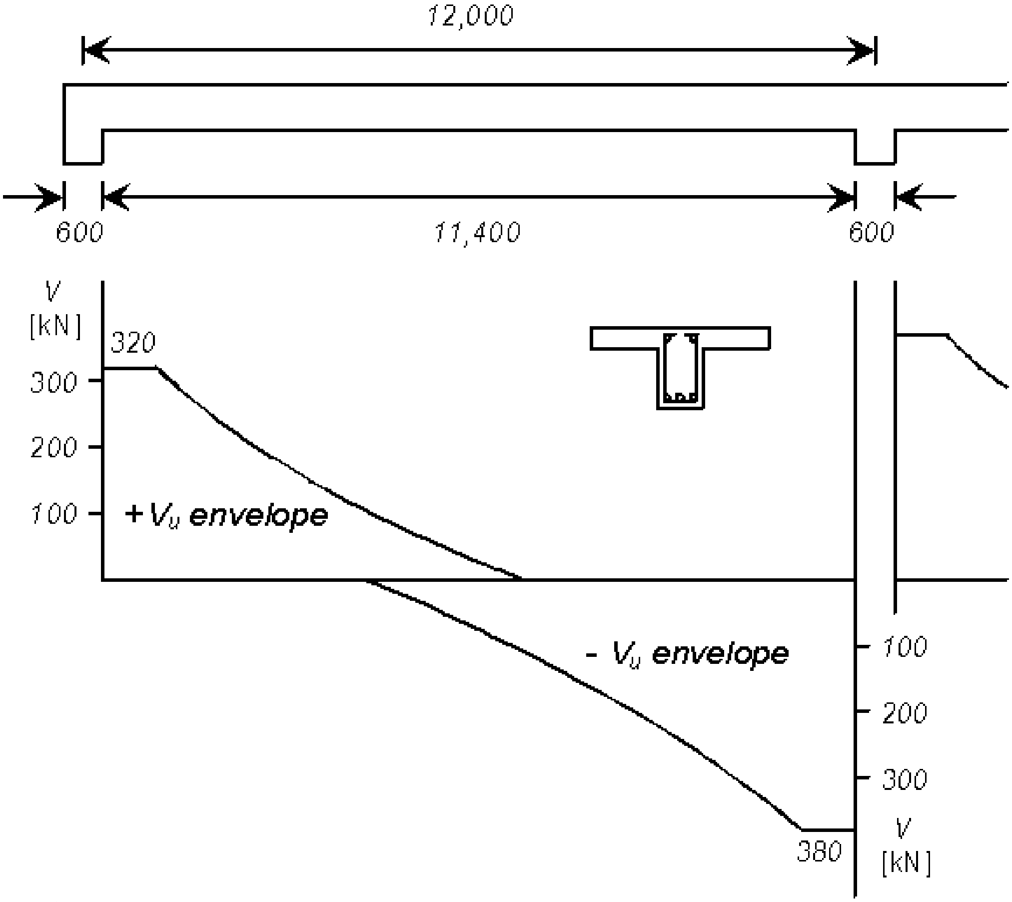
$$\rightarrow req'd\ s = \frac{\phi A_v f_y d}{req'd\ \phi V_s} \qquad \text{max } s = \frac{d}{4} \text{ or } 300 \text{ mm}$$

(5) $\phi V_c + \phi\left(\dfrac{2}{3}\sqrt{f_{ck}}\right) b_w d < V_u$ → *Increase the beam size !*

◉ Example 6-2 : 콘크리트구조설계기준(2007)에 따른 Shear Design

b_w = 460 mm d = 800 mm f_{ck} = 24 MPa

SD 300, D13 U-Stirrup → $A_v = 2A_b$ = 253 mm^2



1. Basic Calculation

$$\phi V_c = \phi\left(\frac{1}{6}\sqrt{f_{ck}}\right)b_w d = 0.75\left(\frac{1}{6}\right)\sqrt{24}\,(460)(800)/1000 = 225\ \text{kN}$$

$$\frac{1}{2}\phi V_c = \frac{1}{2}(225) = 112.5\ \text{kN}$$

$$\phi\left(\frac{1}{3}\sqrt{f_{ck}}\right)b_w d = 0.75\left(\frac{1}{3}\right)\sqrt{24}\,(460)(800)/1000 = 450\ \text{kN}$$

$$\phi V_c + \phi\left(\frac{1}{3}\sqrt{f_{ck}}\right)b_w d = 225 + 450 = 675\ \text{kN} > \max\ V_u = 380\ \text{kN}$$

$$\therefore\ \max\ s = \frac{d}{2} = 400\ \text{mm}\quad \text{or}\quad 600\ \text{mm}\quad \rightarrow\quad \max\ s = 400\ \text{mm}$$

From min. A_v : $$\max\ s = \frac{A_v f_y}{0.0625\sqrt{f_{ck}}\,b_w} = \frac{253(300)}{0.0625\sqrt{24}\,(460)} = 539\ \text{mm}$$

or $$\max\ s = \frac{A_v f_y}{0.35\,b_w} = \frac{253(300)}{0.35(460)} = 471\ \text{mm}$$ → *Not Critical.*

2. Left End Shear Design

$\max\ V_u = 320\text{ kN} \qquad req'd\ \phi V_s = V_u - \phi V_c = 320 - 225 = 95\text{ kN}$

$$req'd\ s = \frac{\phi A_v f_y d}{req'd\ \phi V_s} = \frac{0.75(253)(300)(80)}{95(1{,}000)} = 479\text{ mm} > \max s$$

Use s = 400 mm : 첫 번째 스터럽을 받침부 내면에서 200 mm 떨어진
위치에 배근 (※ 첫번째 stirrup을 s/2 이내에 배치)

For s = 400 mm, $\phi V_s = \dfrac{\phi A_v f_y d}{s} = \dfrac{0.75(253)(300)(800)}{400(1{,}000)} = 114\text{ kN}$

$$\phi V_n = \phi V_c + \phi V_s = 225 + 114 = 339\text{ kN}$$

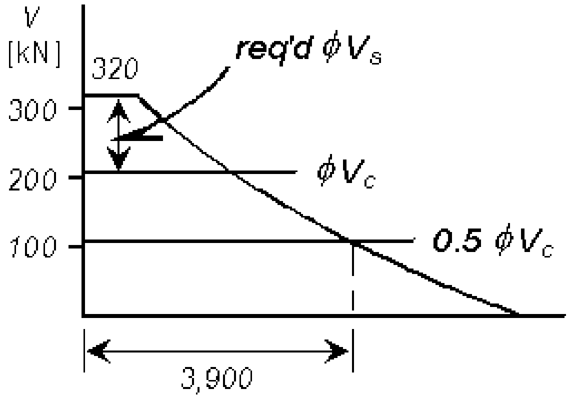
From left face, corresponding point to $\dfrac{1}{2}\phi V_c\ (= 112.5\text{ kN}) \rightarrow$ 3,900 mm

$$req'd\ n = \frac{3{,}900 - 200}{400} + 1 = 10.25$$

∴ *<u>Use 11 stirrups @ 400 mm</u>*

Last stirrup location from left face

= 200 + 10 (400) = 4,200 mm



3. Right End Shear Design

$\max\ V_u = 380\text{ kN}$

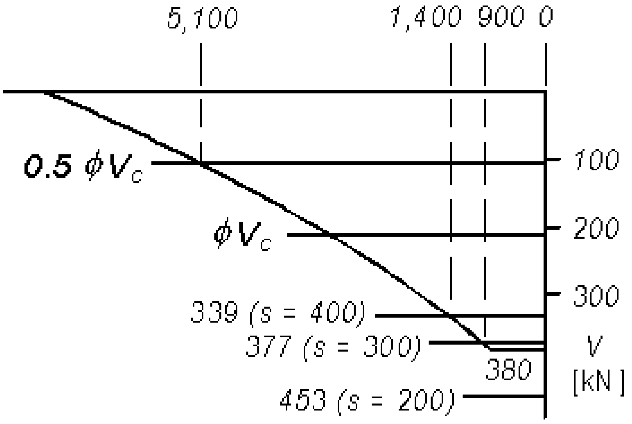
$req'd\ \phi V_s = V_u - \phi V_c = 380 - 225 = 155\text{ kN}$

$$req'd\ s = \frac{\phi A_v f_y d}{req'd\ \phi V_s} = \frac{0.75(253)(300)(80)}{155(1{,}000)} = 294\text{ mm} < \max s$$

$$\phi V_s = \frac{\phi A_v f_y d}{s} = \frac{0.75(253)(300)(800)}{s(1{,}000)} = \frac{45{,}540}{s}\text{ kN}$$

$$\phi V_n = \phi V_c + \phi V_s = 225 + \frac{45{,}540}{s} \text{ [kN]}$$

s [mm]	ϕV_s	ϕV_n
200	228	453
300	152	377
400	114	339



From right face, corresponding point to $\frac{1}{2}\phi V_c \rightarrow$ 5,100 mm

ϕV_n (s = 400) → 1,400 mm

ϕV_n (s = 300) → 900 mm

Try 200 mm & 400 mm spacing

① 첫 번째 스터럽을 받침부 내면에서 100 mm 떨어진 위치에 배근

② s = 200 mm : 8 @ 200 mm → 마지막 위치 = 1,700 mm

③ s = 400 mm : 9 @ 400 mm → 마지막 위치 = 5,300 mm

∴ *Use 18 stirrups @ 200 & 400 mm*

4. Shear Strength Envelope and Drawing