Total = 140 points

Final Exam

Plasticity in Reinforced Concrete

Prof. Jang-Ho Kim
Dept. of Civil & Environment Engineering
2018. 6. 15

1. Show that for any combination of principal stresses, the associated flow rule for Tresca yield criterion gives $\left|\dot{\varepsilon}_1^p\right| + \left|\dot{\varepsilon}_2^p\right| + \left|\dot{\varepsilon}_3^p\right| = \phi$. [20]

2. An elastic-perfectly plastic solid with a uniaxial yield stress of 300MPa is assumed to obey the Tresca yield criterion and its associated flow rule. If the rate of plastic work per unit volume is $1.2 MV/m^3$, find the principal plastic strain-rate components when [30]
   (a) $\sigma_1 = 300MPa,\ \sigma_2 = 100MPa,\ \sigma_3 = 0$ [10]
   (b) $\sigma_1 = 200MPa,\ \sigma_2 = -100MPa,\ \sigma_3 = 0$ [10]
   (c) $\sigma_1 = 200MPa,\ \sigma_2 = -100MPa,\ \sigma_3 = -100MPa$ [10]

3. Derive the Mohr-Coulomb criterion as follows. [45]
   (a) Using the theory of Mohr's circles in plane stress, in particular Eq. (A) and (B), find the direction θ such that $\tau_\theta - \mu(-\sigma_\theta)$ is maximum. [15]

   (A) $\sigma_\theta = \frac{1}{2}(\sigma_1 + \sigma_2) + \frac{1}{2}(\sigma_1 - \sigma_2)\cos 2(\theta - \theta_1)$

   (B) $\tau_\theta = -\frac{1}{2}(\sigma_1 - \sigma_2)\sin 2(\theta - \theta_1)$

   (b) Show that this maximum value is $\left[\sqrt{1+\mu^2}\,|\sigma_1 - \sigma_2|/2\right] + \left[\mu(\sigma_1 + \sigma_2)/2\right]$ and that the Mohr-Coulomb criterion results when this value is equated to the cohesion c with $\mu = \tan\phi$ [15]
   (c) Show that the Mohr circles whose parameters σ_1, σ_2 are governed by this criterion are bounded by the lines $\pm\tau_\theta = \sigma_\theta \tan\phi - c$ [15]

4. Given the yield stress σ_t and σ_c is uniaxial tension and compression, respectively, find the yield stress in shear resulting from the following yield criteria. [45]
   (a) Mohr-Coulomb [15]
   (b) Drucker-Prager [15]
   (c) von Mises [15]